QUANTUM FREQUENCY COMBS

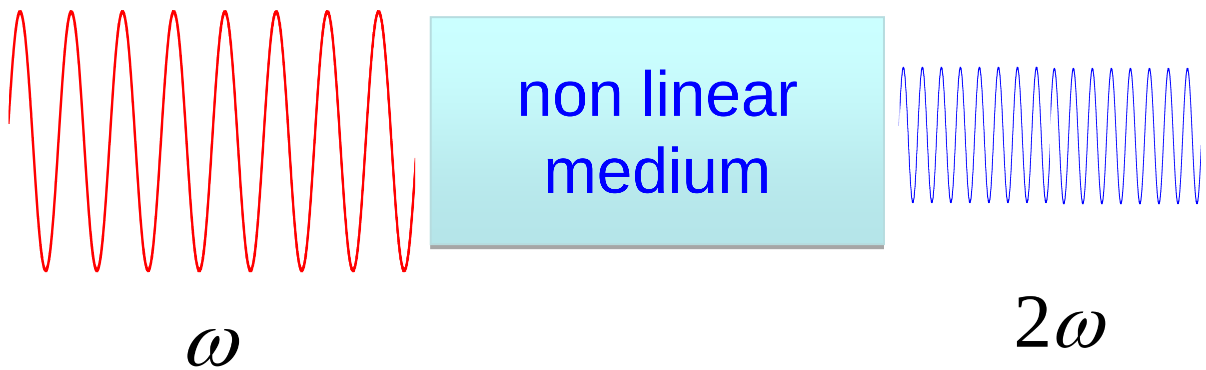
Claude Fabre

Laboratoire Kastler Brossel
Physique quantique et applications

University Pierre et Marie Curie,
Ecole Normale Supérieure
Paris, France

Non linear optics deals essentially with classical light

Second Harmonic Generation

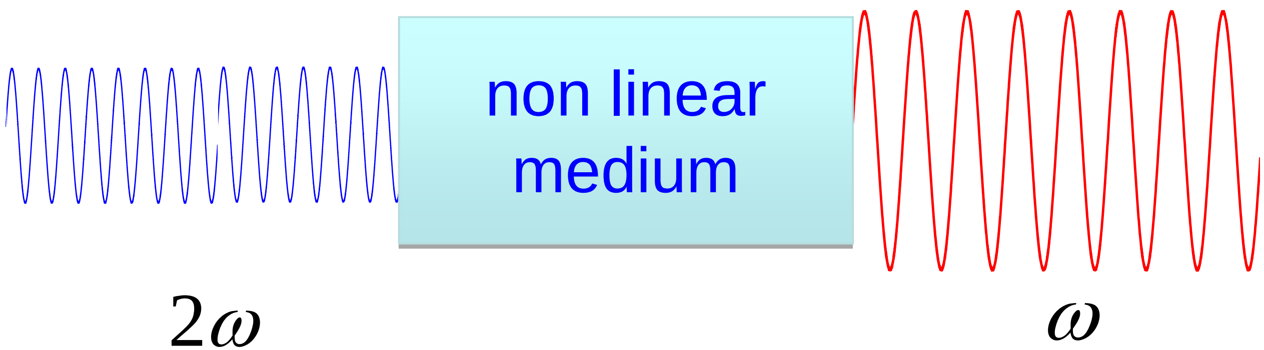


main features described by classical Maxwell equations :

wave in $\longrightarrow$ wave out

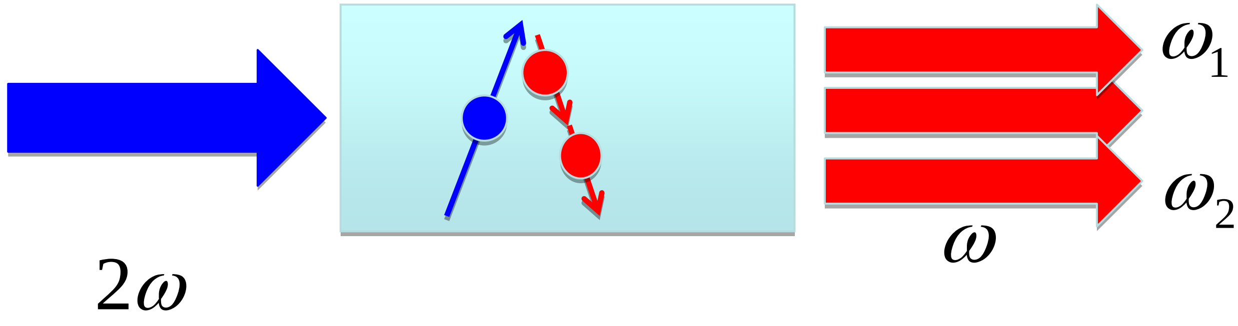
Non linear optics deals essentially with classical light

Parametric Down Conversion



Parametric fluorescence
cannot be described by classical Maxwell equations :
wave in $\longrightarrow$ no wave out !

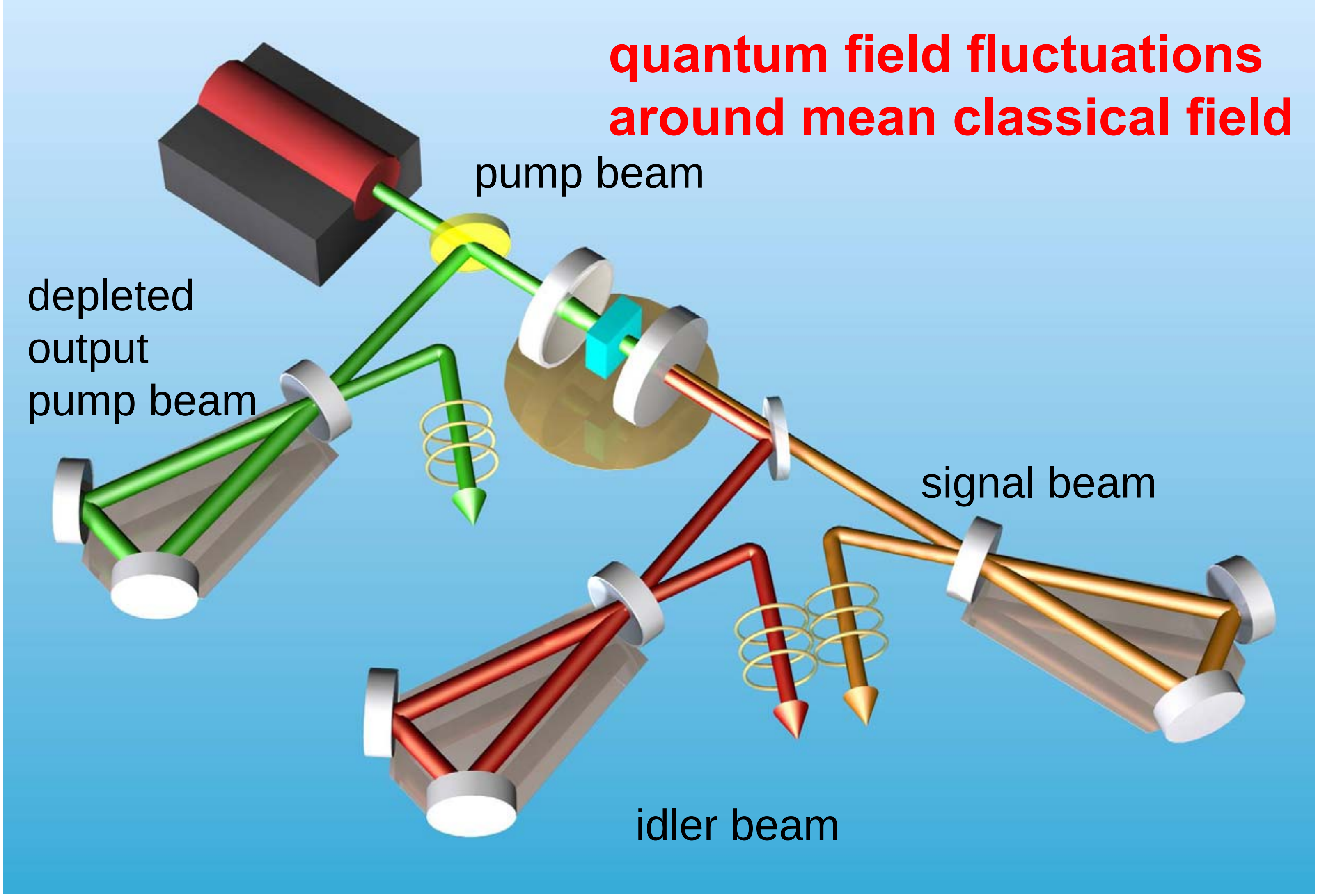
Parametric Down Conversion: a paradigm for quantum optics



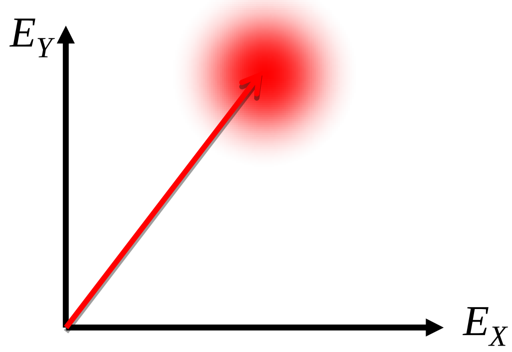
generation of **twin photons**
at the heart of almost all quantum effects
L. Mandel experiments: Friberg et al PRL **54**, 2011(1985)

in the non degenerate case,
it can produce
entangled signal and idler photons

Parametric Down Conversion in a cavity: Properties of quantum fluctuations of the OPO:

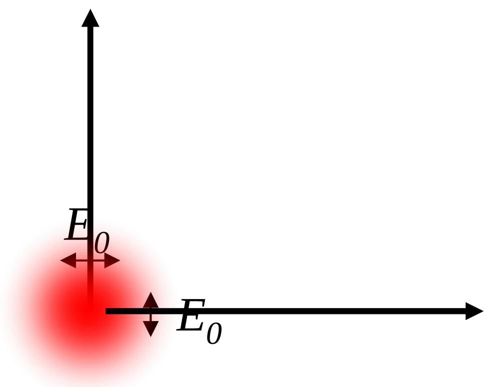


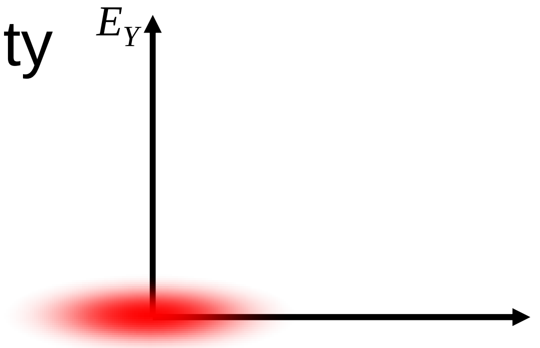
Squeezing of quantum fluctuations



$$\Delta E_X \Delta E_Y \geq E_0^2$$

Heisenberg inequality





vacuum

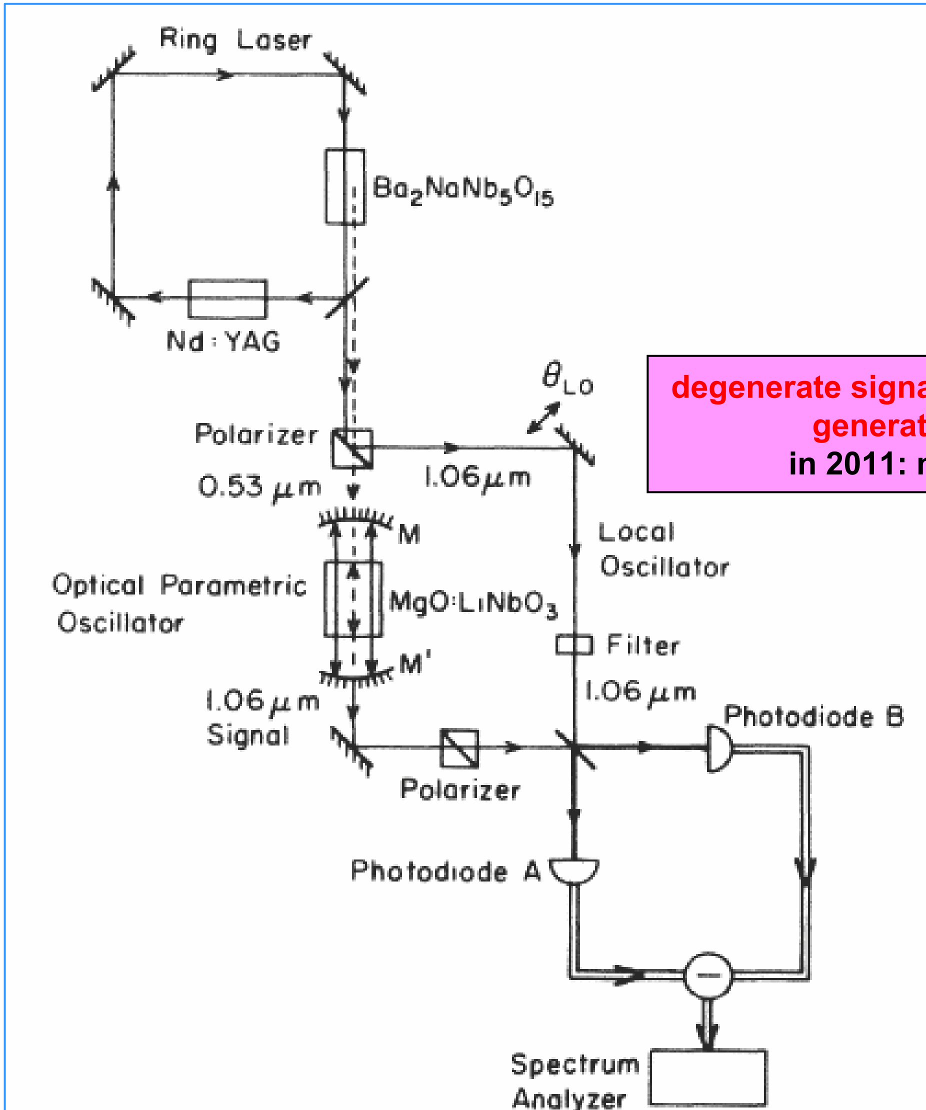
squeezed vacuum

$$\Delta E_X = E_0 \quad \Delta E_Y = E_0$$

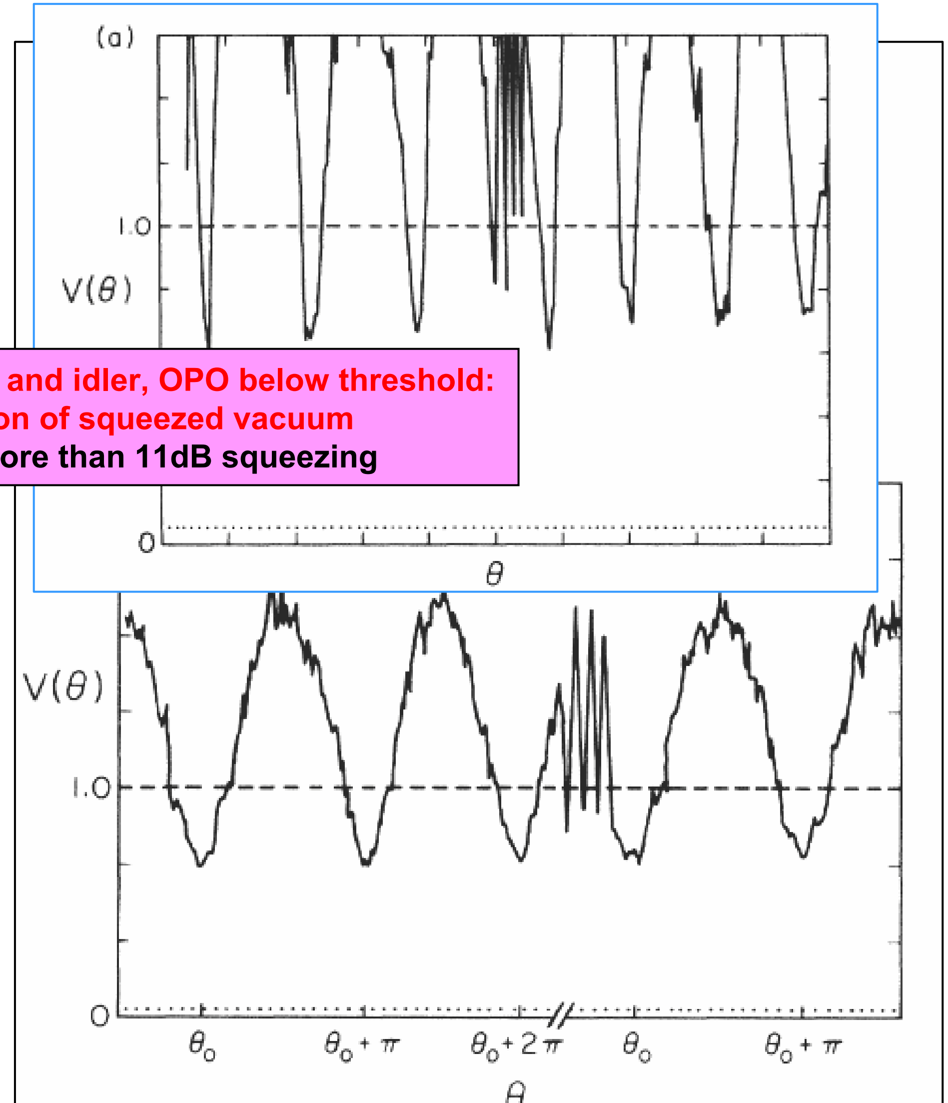
$$\Delta E_X \leq E_0 \quad \Delta E_Y \geq E_0$$

Generation of Squeezed States by Parametric Down Conversion

Ling-An Wu, H. J. Kimble, J. L. Hall,[a] and Huifa Wu
Department of Physics, University of Texas at Austin, Austin, Texas 78712
(Received 11 September 1986)



degenerate signal and idler, OPO below threshold:
generation of squeezed vacuum
in 2011: more than 11dB squeezing



VOLUME 59, NUMBER 22 PHYSICAL REVIEW LETTERS 30 NOVEMBER 1987

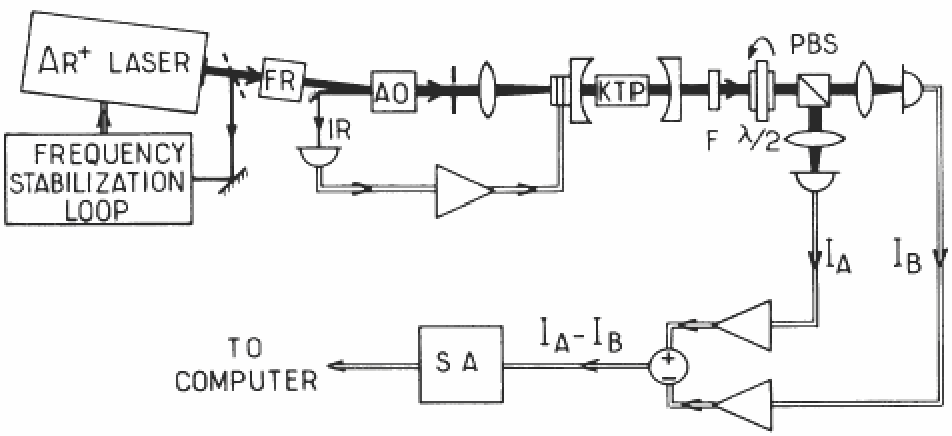
Observation of Quantum Noise Reduction on Twin Laser Beams

A. Heidmann, R. J. Horowicz, S. Reynaud, E. Giacobino, and C. Fabre
Laboratoire de Spectroscopie Hertzienne de l'Ecole Normale Supérieure, Université Pierre et Marie Curie, 75252 Paris Cedex 05, France

and

G. Camy
Laboratoire de Physique des Lasers, Université de Paris Nord, 93430 Villetaneuse, France
(Received 3 August 1987)

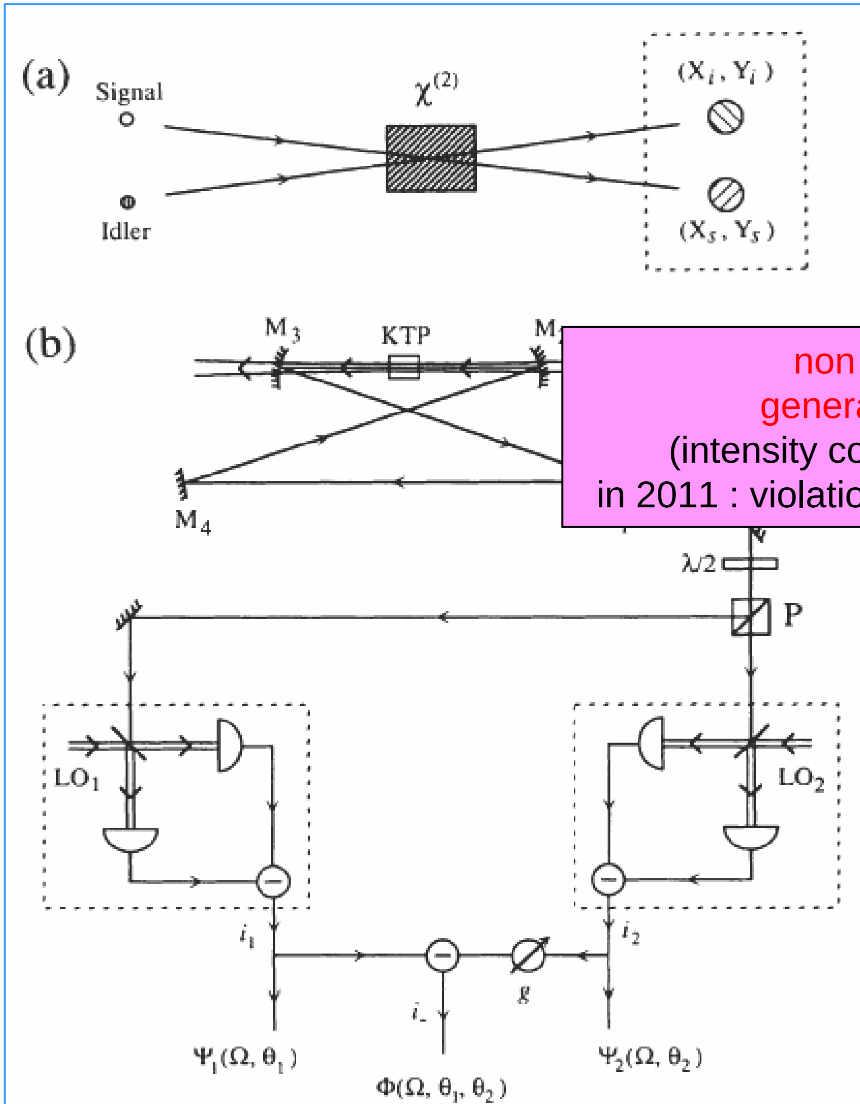
non degenerate signal and idler, above threshold:
generation of "twin beams"
in 2011: 9.9dB of intensity correlations

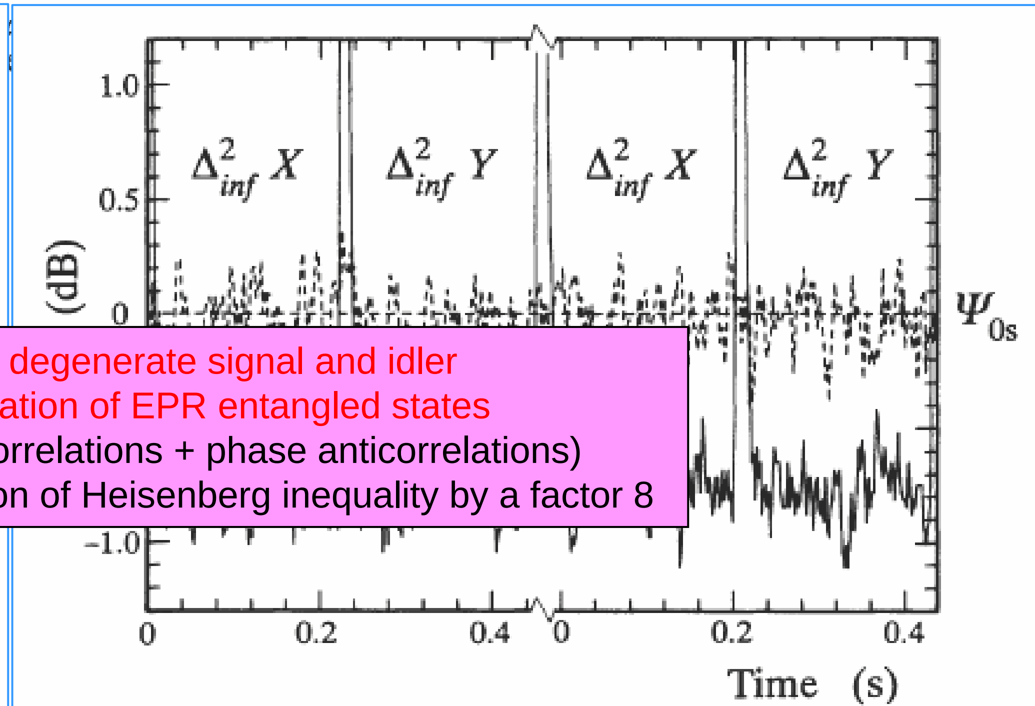


VOLUME 68, NUMBER 25 PHYSICAL REVIEW LETTERS 22 JUNE 1992

Realization of the Einstein-Podolsky-Rosen Paradox for Continuous Variables

Z. Y. Ou, S. F. Pereira, H. J. Kimble, and K. C. Peng[a]





non degenerate signal and idler
generation of EPR entangled states
(intensity correlations + phase anticorrelations)
in 2011 : violation of Heisenberg inequality by a factor 8

$$\Delta^2_{\mathrm{inf}} X \, \Delta^2_{\mathrm{inf}} Y = 0.70 \pm 0.01$$

Europhys. Lett., **40** (1), pp. 25-30 (1997)

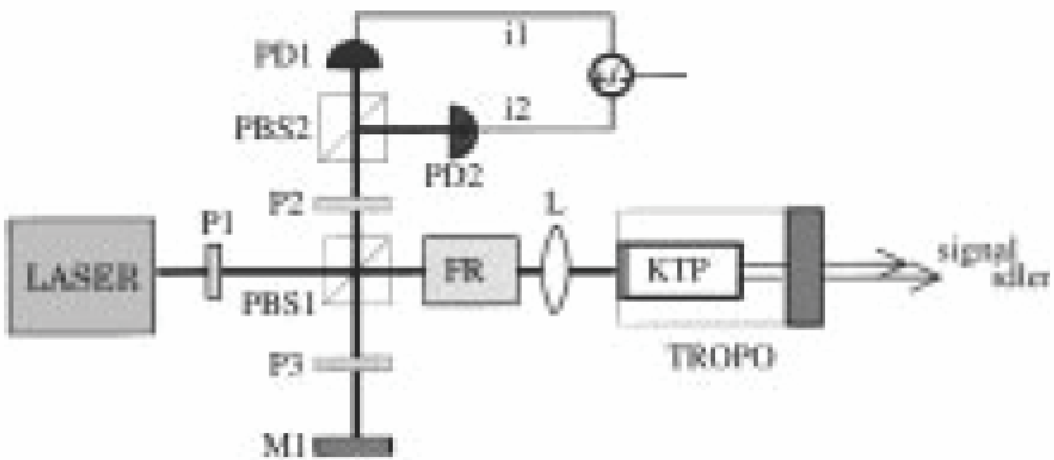
Observation of squeezing using cascaded nonlinearity

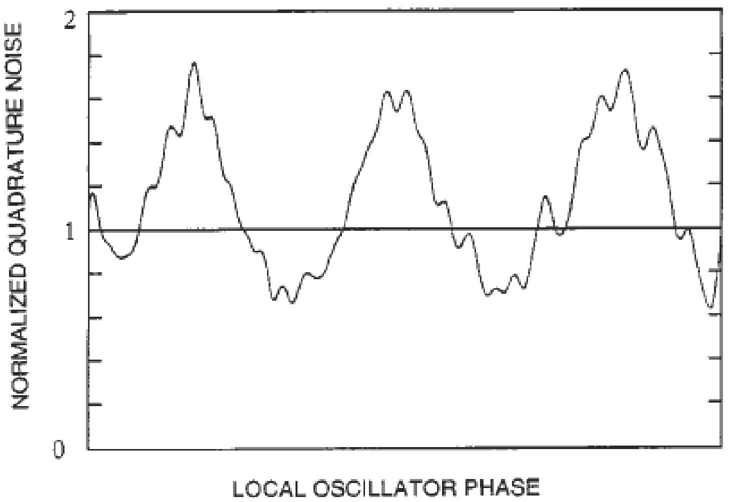
K. Kasai(*), Gao Jiangrui(**) and C. Fabre

Laboratoire Kastler Brossel (***) UPMC - Case 74 75252 Paris Cedex 05, France

(received 20 January 1997; accepted in final form 2 September 1997)

Abstract. – We have observed that the pump beam reflected by a triply resonant optical parametric oscillator, after a cascaded second-order nonlinear interaction in the crystal, is significantly squeezed. The maximum measured squeezing in our device is 30% (output beam squeezing inferred: 48%). The direction of the noise ellipse depends on the cavity detuning and can be adjusted from intensity squeezing to phase squeezing.





Quantum noise reduction effect also
on the depleted pump beam going out of the OPO cavity

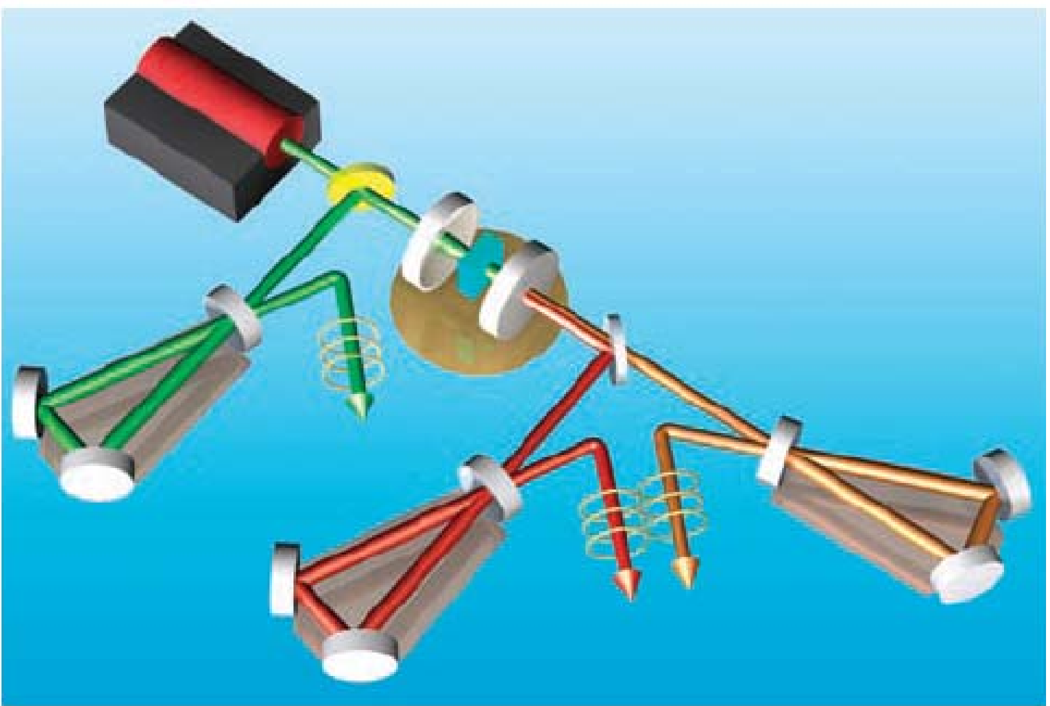
Three-Color Entanglement

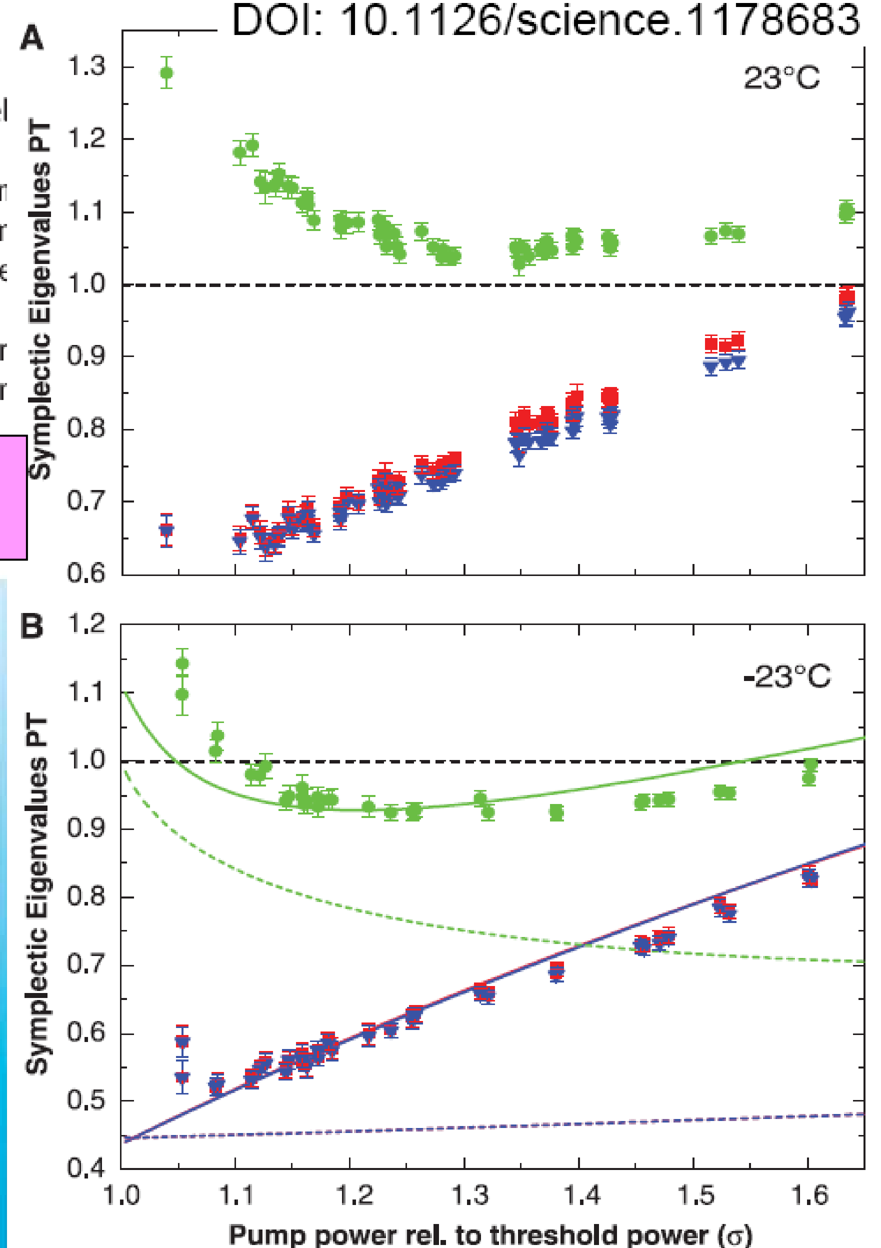
Three-Color Entanglement
A. S. Coelho, *et al.*
Science **326**, 823 (2009);
DOI: 10.1126/science.1178683

A. S. Coelho,[1] F. A. S. Barbosa,[1] K. N. Cassemiro,[2] A. S. Villar,[2,3] M. Martinel

Entanglement is an essential quantum resource for the acceleration of inforr
well as for sophisticated quantum communication protocols. Quantum inform
expected to convey information from one place to another by using entangle
demonstrated the generation of entanglement among three bright beams of
wavelengths (532.251, 1062.102, and 1066.915 nanometers). We also obser
for finite channel losses, the continuous variable counterpart to entanglemer

observation of three partite entanglement between signal, idler and output pump fields





Quantum optics needs
highly multimode quantum states

so far quantum noise reduction and entanglement observed between **single mode fields** (single frequency, TEM_{00})

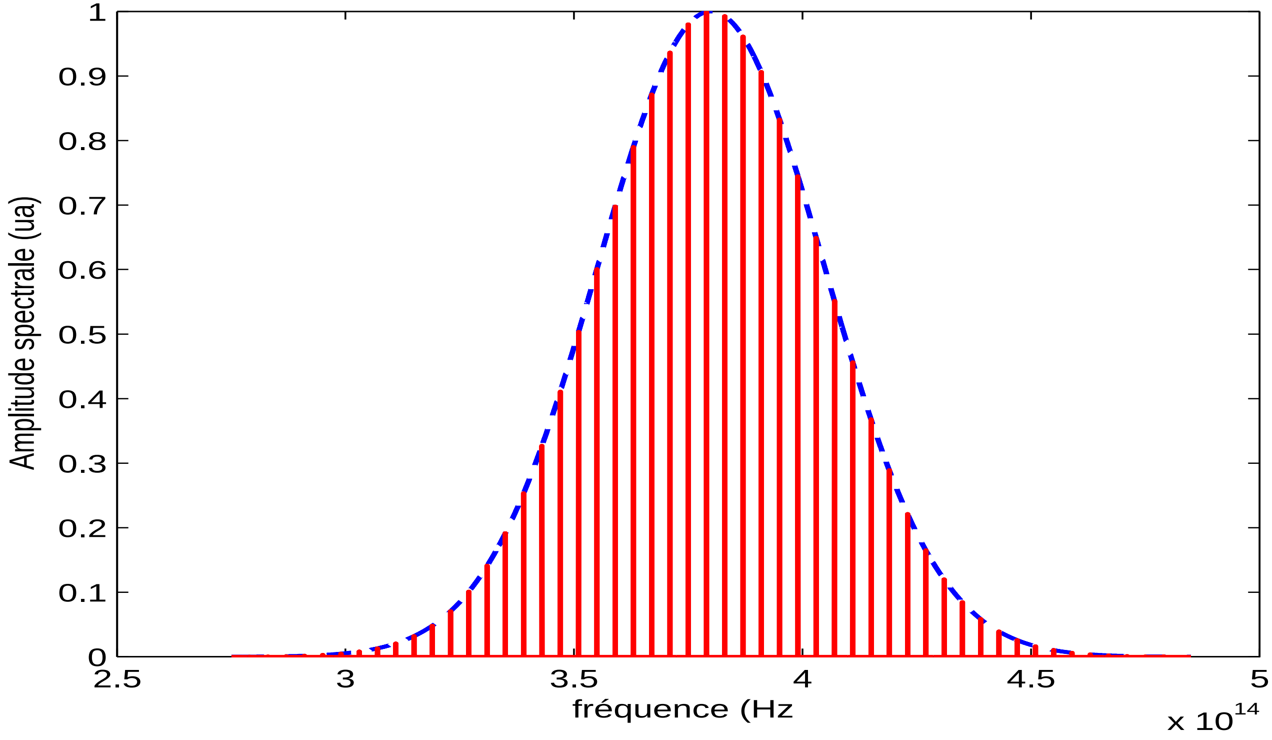
$$\left|\psi\right\rangle = \square\, c_{1,2,....} \left|qubit_1\right\rangle \square \left|qubit_1\right\rangle \square \ldots$$

to generate multi-partite entanglement
needed in quantum computing

$$\psi\rangle = \square\, c_{n_1,n_2....} \left|n_1\ photons \text{in mode} 1\right\rangle \square \left|n_2\ photons \text{s in mode} 2\right\rangle \square \ldots$$

Two degrees of freedom :
- the choice of the quantum state
- the choice of the **mode basis**

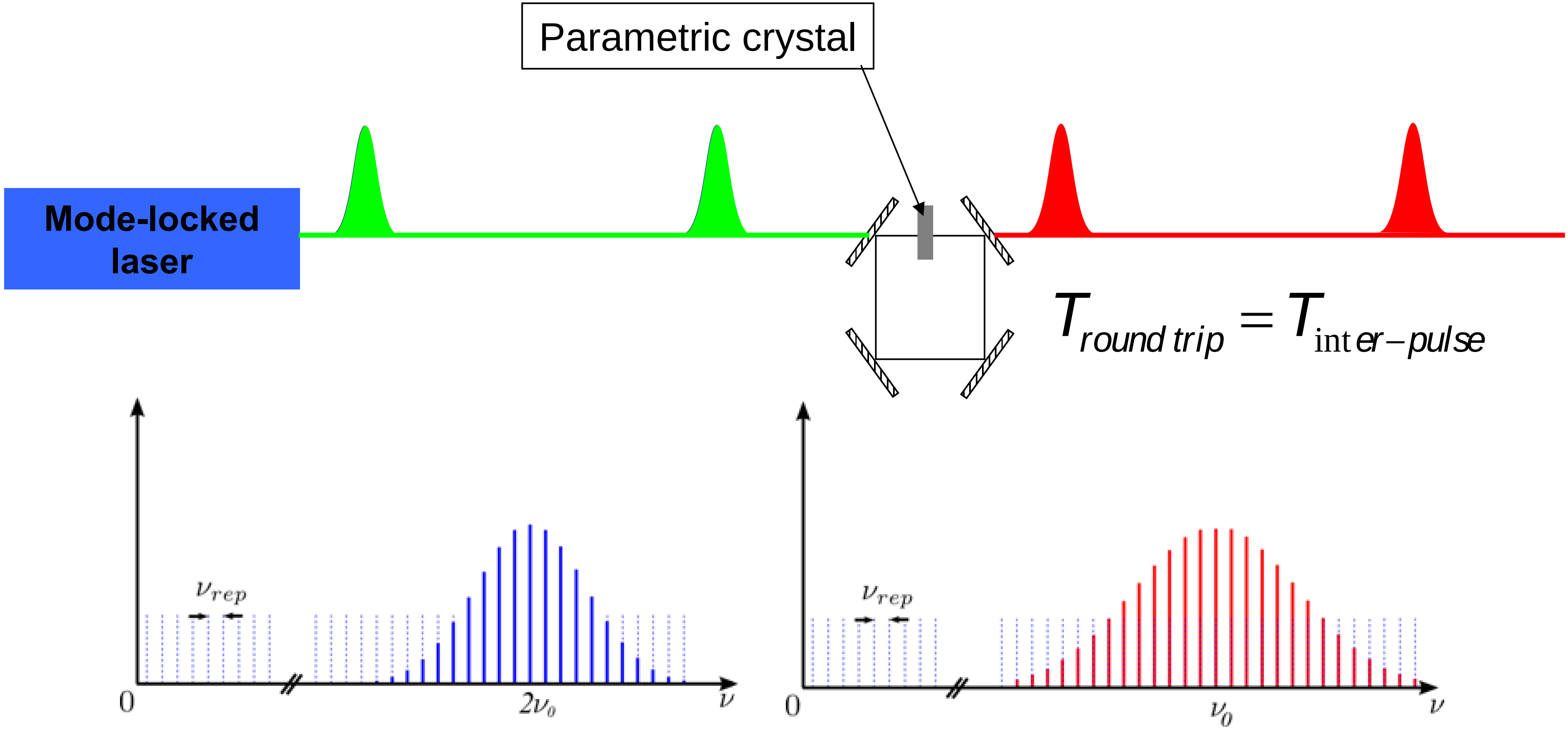
frequency combs : highly multimode objects



trains of 100fs pulses: **10^4 10^5 frequency modes**

Quantum properties
of frequency combs
generated by
parametric down conversion

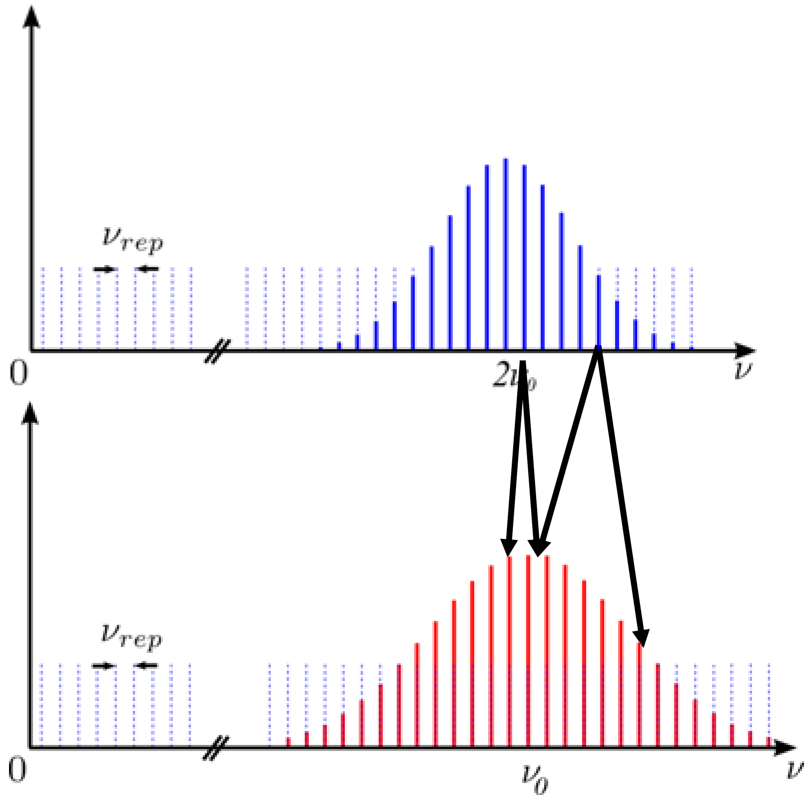
A Quantum Frequency Comb generator: Intracavity parametric down conversion Synchronously Pumped Optical Parametric Oscillator (SPOPO)



Parametric down conversion with multimode pump

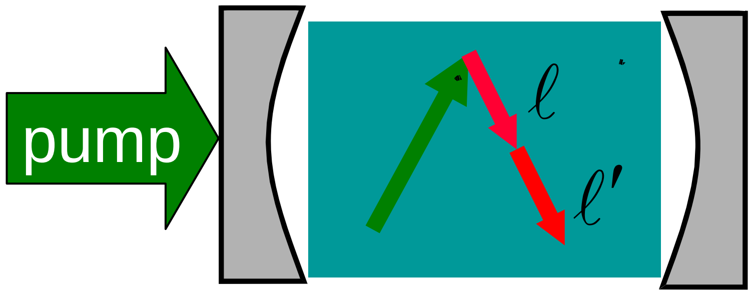
frequency comb
for pump

frequency comb
for signal modes



« twin » photons may have many different fathers !

Quantum description of multi-pump-mode parametric interaction



G. de Valcarcel et al.
*PRA **74** 061801 (2006)*

G. Patera et al.
*EPJD **56** 123 (2010)*

$$\hat{H} = \sum_{\ell,\ell'} \chi(\omega_\ell, \omega_{\ell'}) \; \alpha_{pump}(\omega_\ell + \omega_{\ell'}) \left(\hat{a}_\ell^+ \hat{a}_{\ell'}^+ + \hat{a}_\ell \hat{a}_{\ell'} \right)$$

Crystal phase matching coefficient

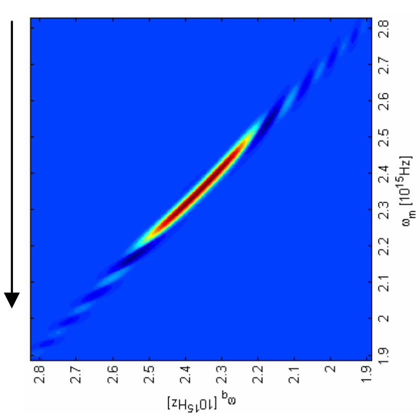
pump spectral amplitude

$$\hat{H} = \sum_{\ell,\ell'} G_{\ell,\ell'} \left(\hat{a}_\ell^+ \hat{a}_{\ell'}^+ + \hat{a}_\ell \hat{a}_{\ell'} \right)$$

Symmetrical matrix

ℓ'

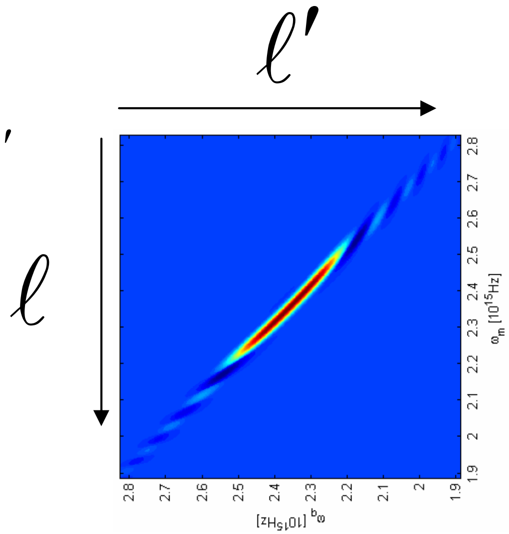
ℓ



$G_{\ell,\ell'}$

Diagonalization of the parametric interaction

$G_{\ell,\ell'}$

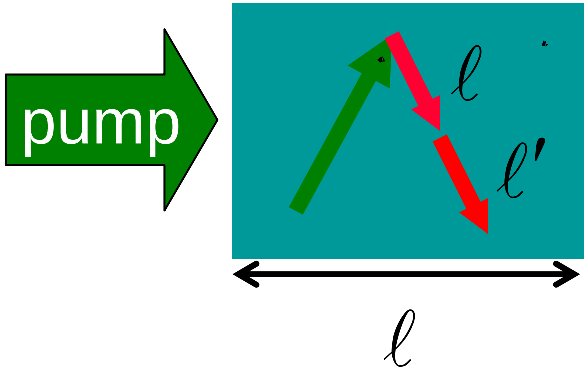


Eigenstates: « supermodes »

$$\hat{b}_k = \sum_{\ell} U_k^{\ell} \hat{a}_{\ell}$$

Eigenvalues: Λ_k $\qquad (|\Lambda_1| \geq |\Lambda_2| \geq .... \geq |\Lambda_k|)$

State generated by single pass through the crystal



$$\hat{H} = \hbar \sum_{k=1}^{N_m} \Lambda_k \left(\hat{b}_k^2 + \hat{b}_k^{+2} \right)$$

$$\left| \Psi_{out} \right\rangle = \prod_{k=1}^{N_m} e^{-i\ell \Lambda_k \left(\hat{b}_k^2 + b_k^{+2} \right)/c} \left| 0 \right\rangle$$

squeezing transformation

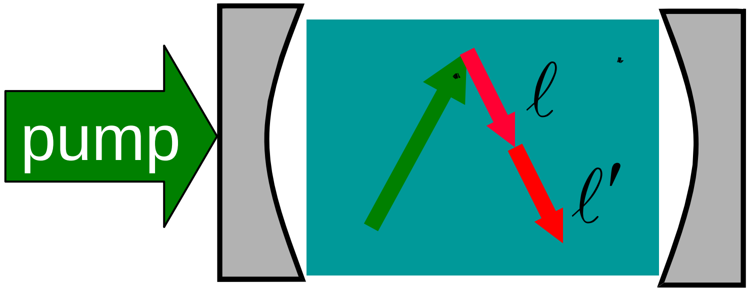
$$\left| \Psi_{\text{out}} \right\rangle = \left| Squeezed\ state_k(\Lambda_1) \right\rangle \otimes \ldots \otimes \left| Squeezed\ state_k(\Lambda_{N_m}) \right\rangle \otimes \left| 0 \right\rangle \otimes \ldots$$

Squeezing factors: $\exp\left(\pm \Lambda_k \ell / c \right)$

System generates N_m squeezed states

N_m Number of non-zero eigenvalues of G (rank of the matrix)

Intracavity multimode parametric interaction:



At pump power P_{th} the system has an **oscillation threshold**

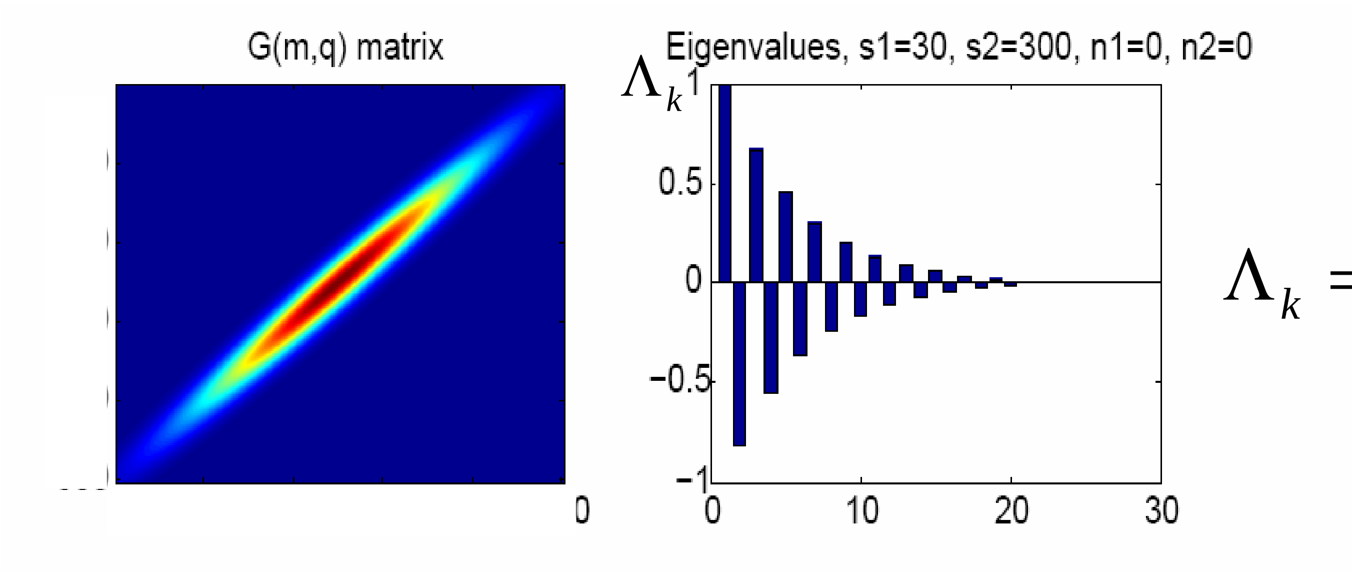
Just below threshold, mode 1 is **perfectly squeezed**

Other modes have squeezing factor:

$$S_k = \left(\frac{|\Lambda_1| - |\Lambda_k|}{|\Lambda_1| + |\Lambda_k|} \right)^2$$

Eigenvalues and eigenmodes

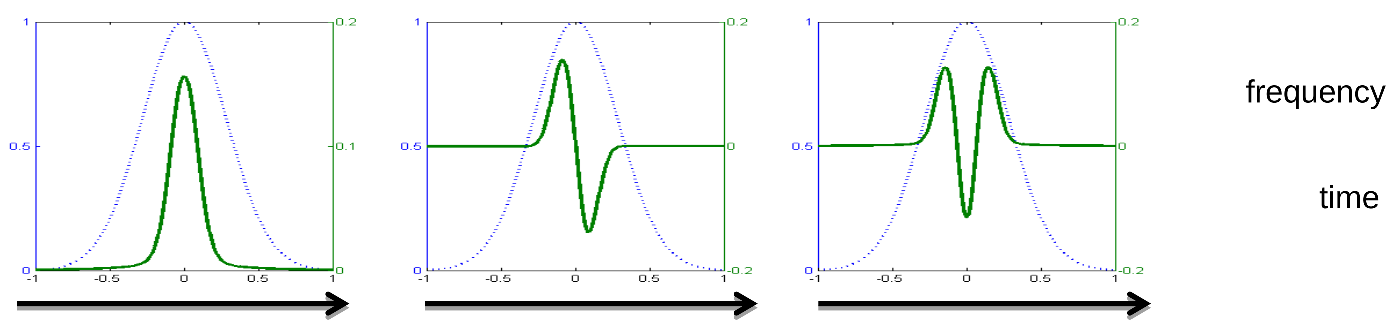
Simple example: Gaussian variation of $G_{\ell,\ell'}$



$$\Lambda_k = \Lambda_0 \left(-r\right)^k$$

Λ_k eigenvalues : 30 non negligible, 99 970 almost zero

Eigenmodes: trains of Hermite-Gauss pulses



From a highly multimode situation

(100,000 frequency modes)

it is in general possible

to extract a smaller Hilbert space

(1 – 100)

in which the quantum effects

are concentrated and maximized

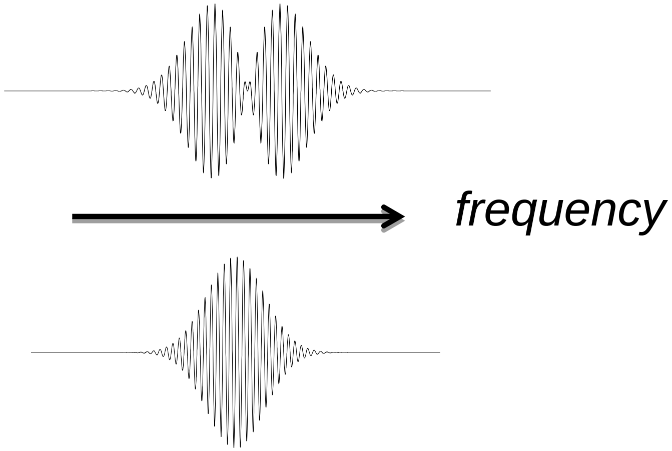
(entanglement and/or squeezing)

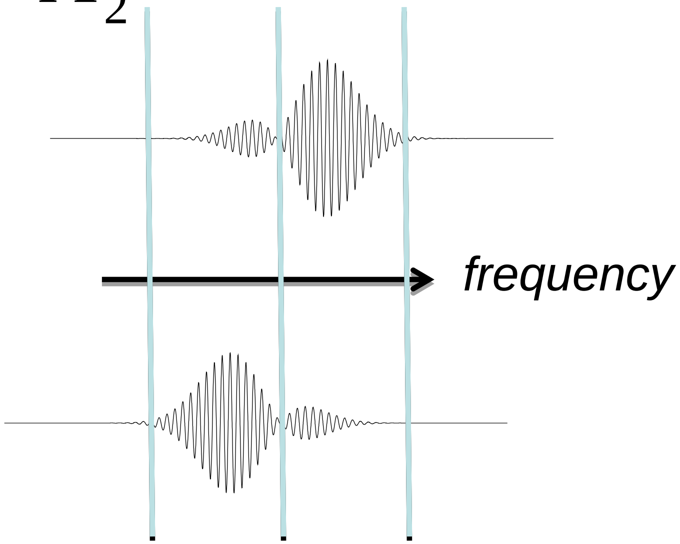
what about entanglement ?

Starting from two squeezed supermodes $v_1(t)$ $v_2(t)$

the mixed modes $v_\pm(t) = \frac{1}{\sqrt{2}}\left(v_1(t) \pm v_2(t)\right)$

are EPR entangled whatever Λ_1 Λ_2





squeezed

entangled

quantum correlations
between different spectral parts of the comb

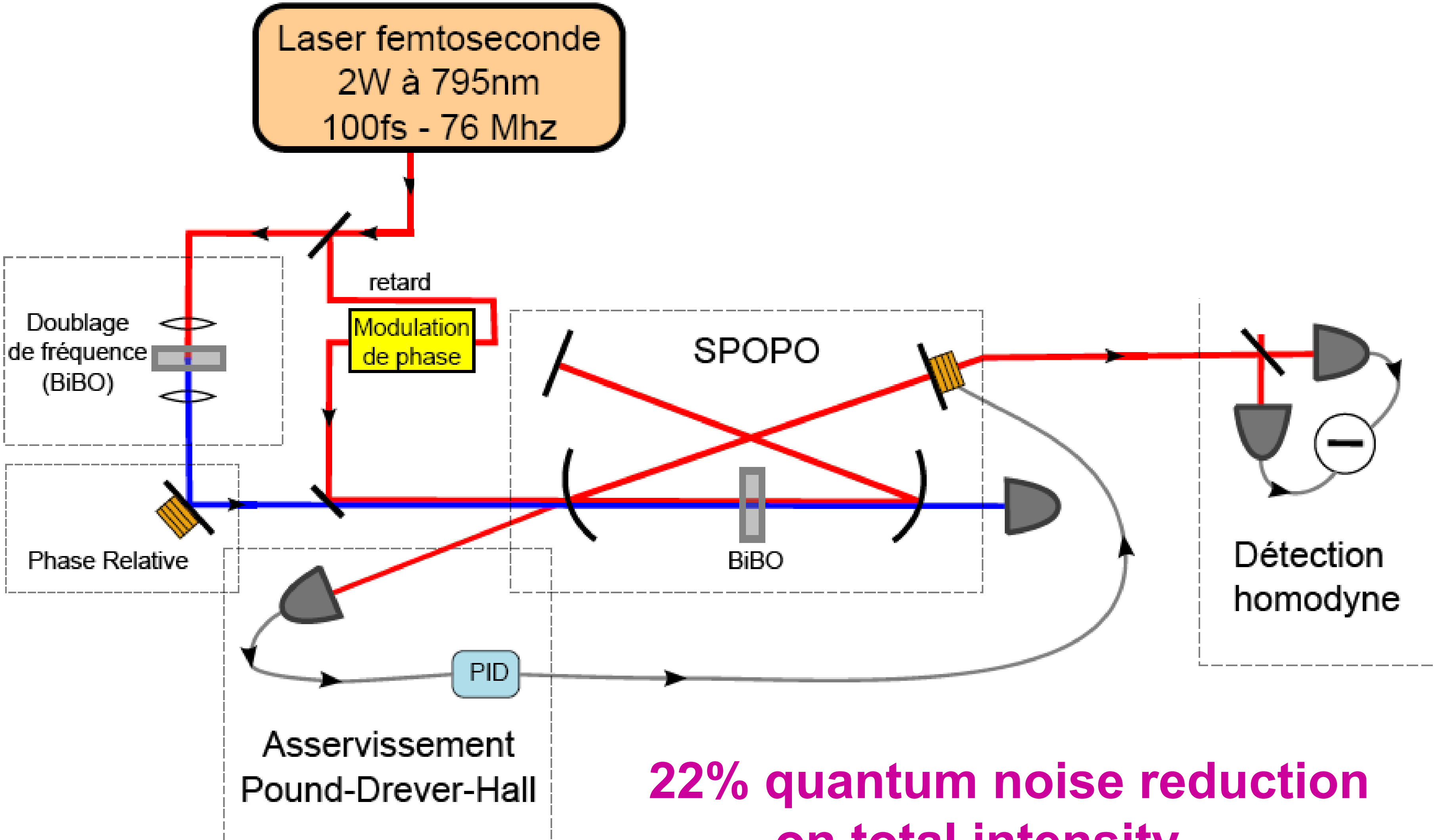
Experiments

(preliminary)

arXiv:1103.6123

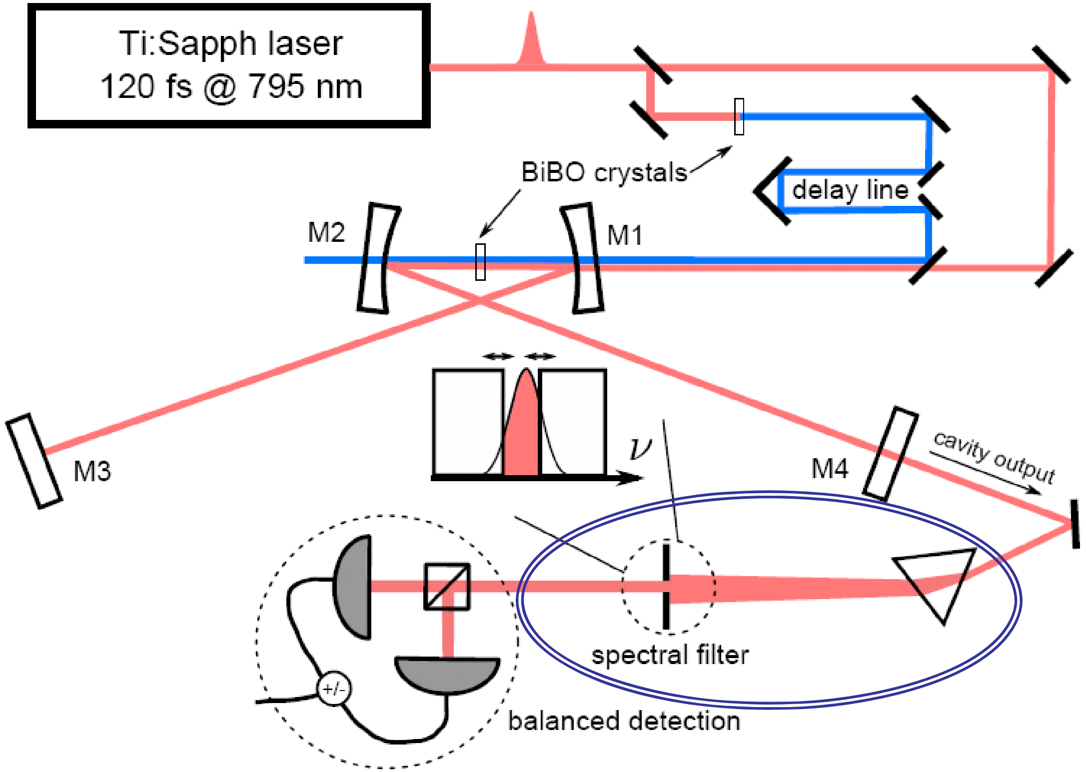
Generation and characterization of multimode quantum frequency combs

O. Pinel, Pu Jian, R. Medeiros, Jingxia Feng, B. Chalopin, C. Fabre, N. Treps

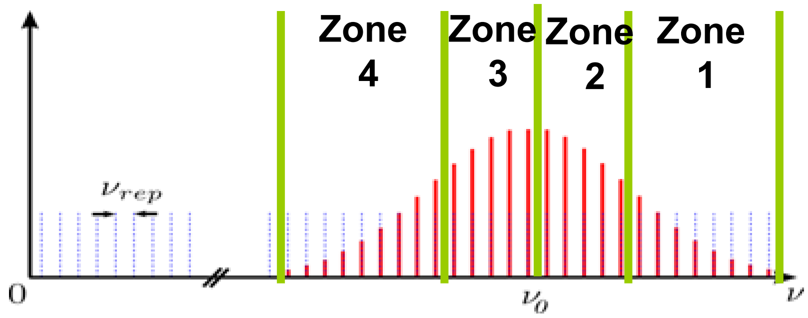


22% quantum noise reduction on total intensity

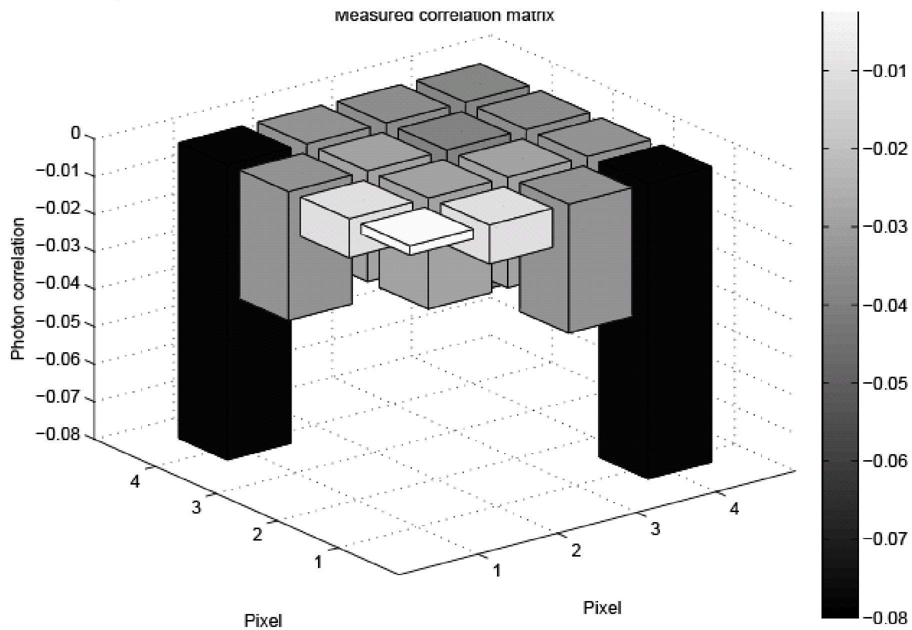
Evidence for multimode squeezing in SPOPO: spectral analysis of the generated light



Existence of quantum correlations between different frequency zones of the SPOPO



Experimental intensity correlation matrix



Eigenmodes of the correlation matrix:
Mode basis of uncorrelated states

squeezed

P=96%

σ^2=0.86

anti-squeezed ?

P=1%

σ^2=1.06

squeezed

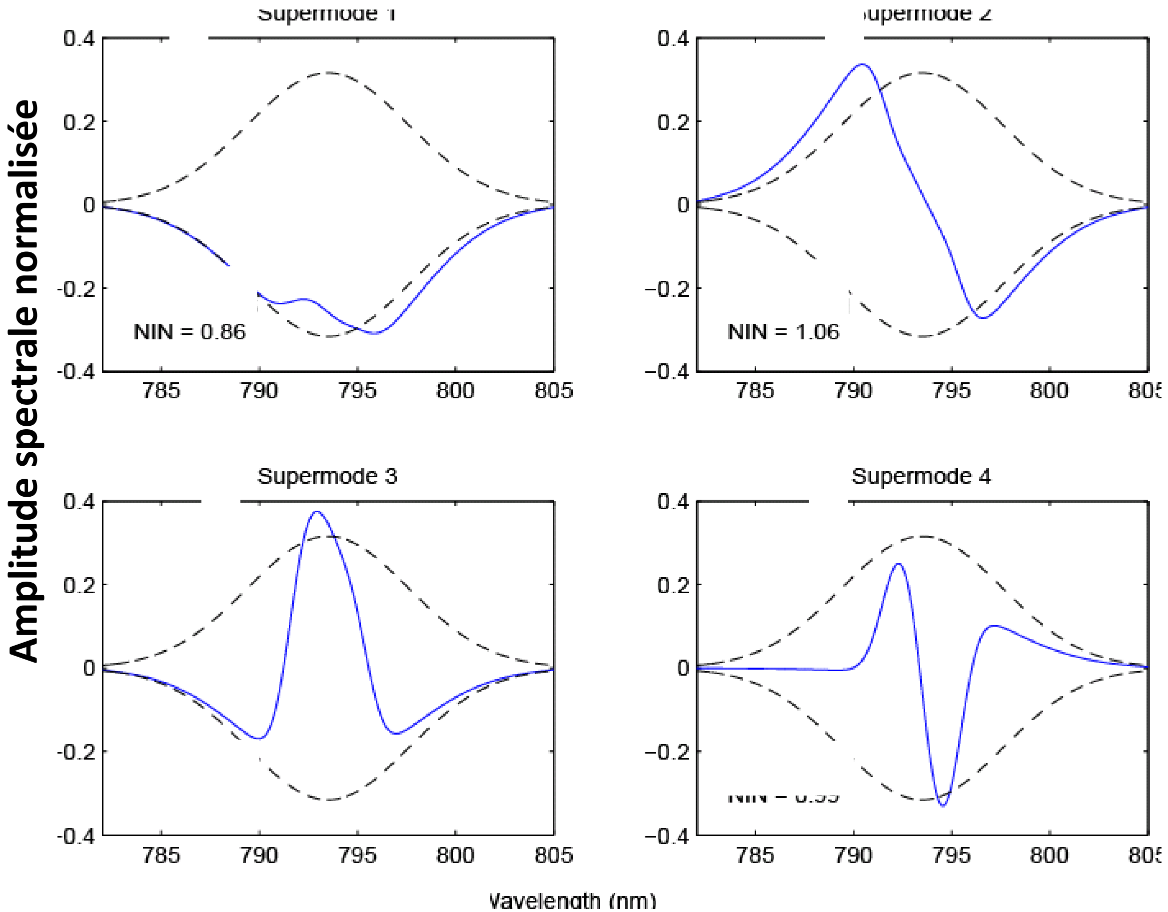
P=3%

σ^2=0.98

vacuum

P=0%

σ^2=0.99



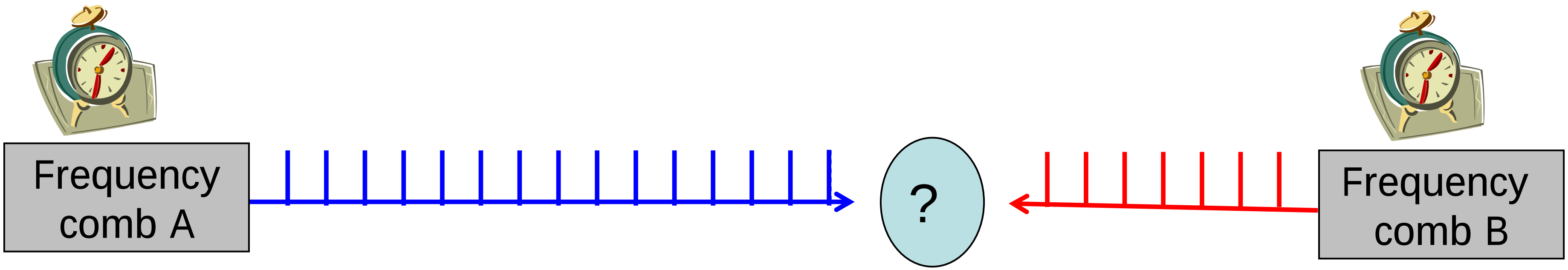
Longueur d'onde (nm)

precise characterisation of the different non-classical modes
of the quantum frequency comb

Application to Quantum Metrology of time

frequency combs : clocks

synchronization
of remote clocks



- What is the best possible estimation of a time delay between the two clocks ?
- How to make such a measurement ?

general limit of sensitivity in parameter estimation

- optimized over all possible measurements
- optimized over all possible data processing strategies

Quantum Cramer Rao Bound

*Helstrom, Phys Lett **A25**, 1012 (67)*
*Braunstein, Caves Phys Rev Lett **72**, 3439 (94)*

for Gaussian pulses in a coherent state :

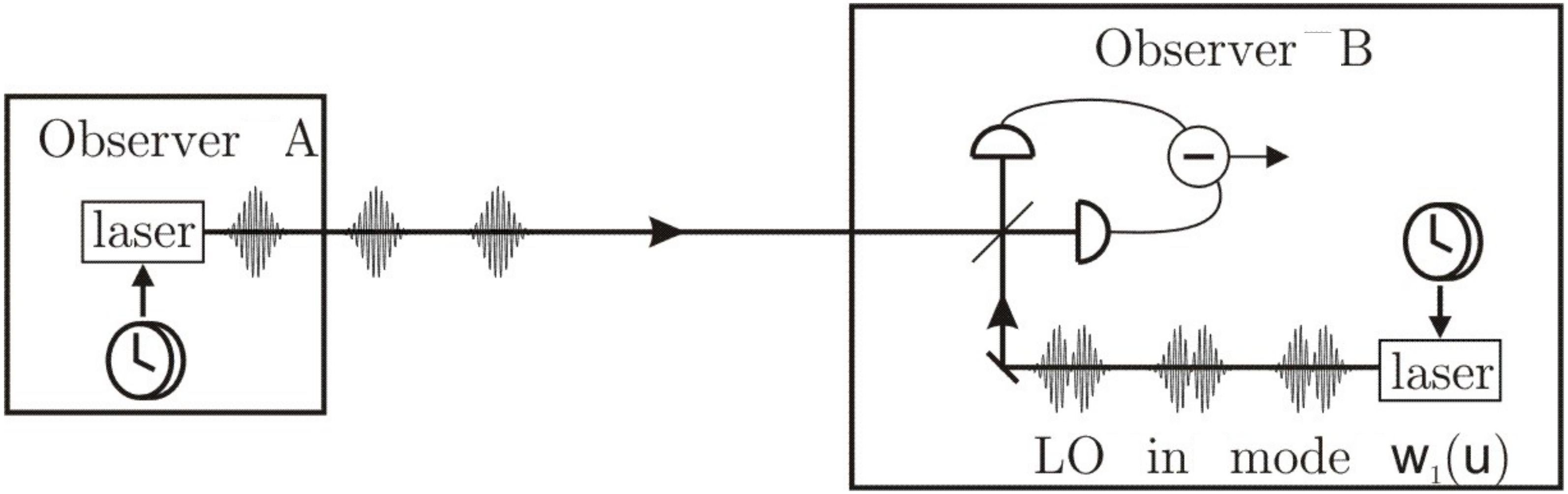
$$(\Delta t)_{S-CRb} = \frac{1}{\sqrt{N}} \frac{1}{2\sqrt{\omega_0^2 + \Delta\omega^2}}$$

B. Lamine, C. Fabre, N. Treps,
*Phys. Rev. Letters **101** 123601 (2008)*

N : total number of photons
ω_0 : mean frequency
$\Delta\omega$: frequency spread

10mW, 20 fs **:100-yoctosecond range**

How to reach the Quantum Cramer Rao bound ?



Frequency comb A : Gaussian "TEM_{00}"
Frequency comb B : Gaussian "TEM_{10}"

No other measurement technique, using the same state of light, can be more accurate

Can be improved below shot noise
by using multimode non-classical state
such as the one produced by a SPOPO

Conclusion

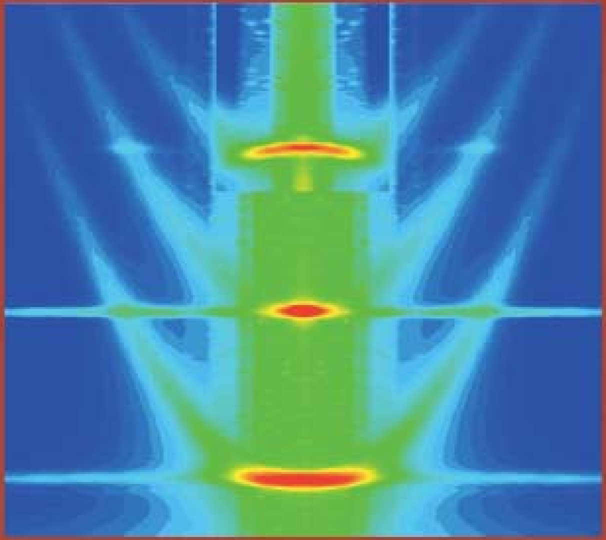
- Quantum Optics, more precisely
  quantum noise reduction and correlation effects
  are another important avatar of NonLinear Optics

- SPOPOs produce multimode non-classical states

- quantum states of light
  likely to optimize sensitivity
  of optical measurements
  especially metrology of time

Introduction to QUANTUM OPTICS

From the Semi-classical Approach to Quantized Light



Gilbert Grynberg, Alain Aspect
and Claude Fabre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Pu Jian
Renné Medeiros