SELF-FOCUSING AND SELF-COMPRESSION OF ELECTROMAGNETIC WAVES IN PLASMA:

BASIC IDEAS AND NEW RESULTS

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Two first publications on self-focusing were based on the consideration of spatial distribution of electrons in the field of an HF electromagnetic wave.

$$\vec{E} = \vec{E}(\vec{r})e^{i\omega t}, \quad \vec{H} = \vec{H}(\vec{r})e^{i\omega t} \quad \vec{F} = -\nabla\Phi_j \; \Phi = \frac{e^2}{4m\omega^2}\left|\vec{E}(\vec{r})\right|^2$$

A.V. Gaponov,
M.A. Miller, 1958

$$N_e = N_{e0}\exp\left(-\frac{\Phi}{kT}\right) = N_{e0}\exp\left(-\frac{\left|E(\vec{r})\right|^2}{E_p^2}\right)$$

H.A.H. Boot,
S.A. Self,
R.B.R. Harvie, 1958

Idea of self-focusing: waveguiding due to compensation diffraction divergence due to arising gradient of the medium properties

G.A. Askaryan (1962)

The steady-state distribution of self-trapped TE polarized plane-waveguide in plasma was found (spatial soliton)

V.I. Talanov (1964/62)

“Self-trapping of optical beams”
The simplest type of self-trapped axisymmetric waveguide was found with a power independent on its size (critical power)

R.Y. Chiao, E. Garmire,
C.H. Townes (1964):

Starting from paper by R.Y. Chiao et al.,
the major research interest was concentrated in nonlinear optics.

The Institute of Applied Physics team
(initially, up to 1977 – Radiophysical Research Institute) –
V.I. Talanov, with participation of V.I. Bespalov, A.G. Litvak, and S.N. Vlasov –
was developing the theory of self-focusing and self-modulation
of a weakly nonlinear medium.

At the same time, we continued the detailed study of various self-action effects in the rarefied and dense (above-critical) plasma:

- self-trapping in rarefied plasma and nonlinear transparency of dense plasma (including 1D models)
- self-focusing of paraxial wave beams in weakly and completely ionized isotropic and magnetized plasma
- envelope solitons and pulse self-compression
- filamentation and modulation instabilities
- self-focusing and self-modulation of relativistically strong em waves
- experimental study in the microwave frequency band

THEORY OF SELF-FOCUSING AND SELF-MODULATION

(1) Nonlinear Parabolic Equation (later called NLSE)

$$E = \frac{1}{2}\left(E(\vec{r})\, e^{-ikz+i\omega t} + \text{c.c.}\right)$$

$$\varepsilon^{NL} = \varepsilon_0\left(1+\varepsilon' f^{N}\left(\left|\bar{E}\right|^2\right)\right), \quad \varepsilon' f^{NL} \ll 1, \quad k = \omega/c\sqrt{\varepsilon_0}$$

Paraxial approximation $k\Lambda_\perp \gg 1, \quad \Lambda_z \sim k\Lambda_\perp^2 \gg \Lambda_\perp; \quad \Lambda_{\perp,z} \sim \dfrac{E}{\left|\nabla_{\perp,z}E\right|}$

$$-2ik\frac{\partial E}{\partial z} + \Delta_\perp E + \kappa^2\varepsilon' f^{NL}\left(\left|E^2\right|\right) = 0$$

V. Talanov,
JETP Lett., **2**, 138 (1965)

(2) Non-Stationary Field in Dispersive Media $\varepsilon_0 = \varepsilon_0(\omega)$

$$-2ik\frac{\partial E}{\partial z} + \Delta_\perp E - k\frac{dV_g}{d\omega}\frac{\partial^2 E}{\partial \tau^2} + k^2\varepsilon' f^{NL}\left(\left|E\right|^2\right)E = 0$$

$V_g = \dfrac{\partial\omega}{\partial k}$ - group velocity, $\quad \tau = z - V_g t$

Nonlinear Parabolic Equation **(NSE)**

A.G. Litvak, V.I. Talanov,
Radiophys. Quantum
Electron., **10**, 296 (1967)

(3) Solitary waves of envelopes

L.A. Ostrovsky, JETP, **24**, 1797 (1967)
A.G. Litvak, V.I. Talanov, JETP, **24**, 797 (1967)

(4) Filamentation Instability

V.I. Bespalov, V.I. Talanov, JETP Lett., **3**, 307 (1966)

Growth rate of spatial perturbations of the plane wave

$$\Gamma = \frac{1}{2}\sqrt{2\varepsilon' k^2 E_0^2 - k_\perp^2}$$

E_0 – plane wave amplitude, $\kappa_\perp$ – spatial frequency of the perturbation

$$G_{\max} = \frac{1}{2} k\varepsilon' E_0^2, \quad k_{\perp opt} = k\sqrt{\varepsilon'} E_0$$

(5) Modulational Instability takes place if $\frac{dV_g}{d\omega}\varepsilon_2' > 0$

$$\Omega_{opt} = kV_g\sqrt{\frac{\varepsilon' E_0}{kv_\omega}}$$

A.G. Litvak, V.I. Talanov, (1967)

(6) Spatio-Temporal Modulation

(7) Numerical Calculations of Nonlinear Stage

(8)Ray Optics and Aberration-Free Approximation for Media with Cubic Nonlinearity

$$E = E_0(z)\exp\left(-\frac{r_\perp^2}{2\alpha^2(z)}\right) \quad r_\perp \ll a \quad n=\sqrt{\varepsilon_0+\varepsilon'|E|^2}\cong n_0-\frac{1}{2}n_2 r_\perp^2$$

V. Talanov,
JETP Lett., **2**, 138 (1965)

Assuming that the quasi-Gaussian beam shape is retained (aberration-free)

$$\frac{d^2a}{dz^2}=\frac{1}{a^3}-n_2 a \Rightarrow 1\Big/\left[a^3\left(1-P\big/P^*_{kp}\right)\right], \quad P^*_{kp}=2\pi$$

Blow-up in consistence with computations of NLSE

P.L. Kelly,
PRL, **15,** 1005 (1965)

Steady-state thermal self-focusing

A.G. Litvak,
JETP Lett., **4**, 239 (1966)

$$\varepsilon=\varepsilon_0\left(1+\frac{\partial\varepsilon}{\partial T}\delta T\right), \quad \chi\Delta_\perp(\delta T)=-\alpha|E_0|^2\exp\left(-r_\perp^2\big/a^2(z)\right);$$

$$\delta T=\delta T_0\left(1-\alpha r_\perp^2\right), \quad r_\perp \gg a$$

$$\frac{d^2a}{dz^2}=\frac{1}{a}\left(\frac{1}{a^2}-\beta\right), \quad \beta=\frac{\pi\,{\partial\varepsilon}\big/{\partial T}\,\alpha}{\chi n_0}P$$

Oscillating beam radius

Review paper:
V.I. Bespalov, A.G. Litvak, V.I. Talanov, *Self-action of electromagnetic waves in cubic isotropic media*, in: “Nonlinear Optics”, Nauka, Novosibirsk, 428-462 (1968) (submitted October 1966) [in Russian]

(9) Method of moments

$$-i\frac{\partial\psi}{\partial z}+\Delta_\perp\psi+|\psi|^2\psi=0$$

S.N. Vlasov, V.A. Petrishev, V.I. Talanov, Quant. Electron. Radiophys., **14**, 1062 (1971)

$$a_{eff}^2=\frac{1}{N}\int_{\delta_\perp}(\vec{r}_\perp-\vec{r}_c)^2|\psi(\vec{r}_\perp)|^2dS_\perp,\quad N=\int_{\delta_\perp}|\psi(r_\perp)|^2dS_\perp$$

$$H=\int_{\delta_\perp}\left(|\nabla_\perp\psi|^2-\frac{1}{2}|\psi|^4\right)dS_\perp=const$$

$$\frac{d^2a^2}{dz^2}=H,\quad a^2=a_{eff}^2(0)+Bz+Hz^2,$$

$H<0$ $\quad a_{eff}^2(z=z^*)=0$ - singularity formation

Power going into foci is equal to $P_{critical}$.

The field in the vicinity of nonlinear focus is growing according to the law

$$\psi(0,z)\sim\sqrt{\frac{\ln\left[-\ln\left(z_{sf}-z\right)\right]}{z_{sf}-z}}$$

G.M. Fraiman, JETP, **61**, 228 (1985)

SELF-EFFECTS IN PLASMA

2-Dimensional TE-waveguiding

V. Talanov, 1964

$$E_y = \tilde{E}(x)\exp i(\omega t - kz)$$

$$\frac{d^2E}{dx^2} + k_0^2\left(\varepsilon\left(|E|^2\right) - \gamma^2\right)\vec{E} = 0; \quad k_0 = \frac{\omega}{c},\ \gamma = h/k_0$$

$$\varepsilon = 1 - qe^{-E^2},\ E = \tilde{E}\big/E_p,\ a = \omega^2\big/\omega^2,\ \omega^2 = \frac{4\pi e^2 N_0}{m}$$

$$\gamma^2 = {1 - q\left[1 - \exp\left(-E_m^2\right)\right]}\big/{E_m^2} > 1 - q = \varepsilon_\infty$$

$$E_m^2 \ll 1 \quad E = E_m \quad \cosh^{-1}\left[k_0\sqrt{\gamma^2 - (1-q)}\,x\right]$$

$$\gamma^2 = 1 - q + \frac{1}{2}qE_m^2$$

Above-critical density $q > 1$: self-channeling, when $E_m^2 > E_{thresh}^2 = \dfrac{2(q-1)}{q}$

2-Dimensional TM-waveguiding

A.G. Litvak,
Sov. Radiophysics,
9, 404 (1966);
A.G. Litvak, G.M. Fraiman,
JETP, **44**, 640 (1975)

$$E_x = E_x(x)\exp(i\omega t - ihz), \quad E_z = iE_z(x)\exp(i\omega t - ihz)$$

$$(1) \qquad \left[k_0^2\varepsilon(E^2) - h^2\right]E_x - h\frac{dE_z}{dx} = 0,$$

$$(2) \qquad \frac{d^2E_z}{dx^2} + \kappa_0^2\varepsilon(E^2)E_x + h\frac{dE_z}{dx} = 0, \quad E^2 = E_x^2 + E_z^2 \ll 1$$

$$\varepsilon = 1 - q^2 + q^2E^2 = \varepsilon_\infty + q^2E^2;$$

weak
field

$$E^2 \ll 1 \quad \gamma_{TM}^2 \sim 1 - q^2 + {q^2E_m^2}/{2} = \gamma_{TE}^2 \qquad \gamma = {h}/{k_0}$$

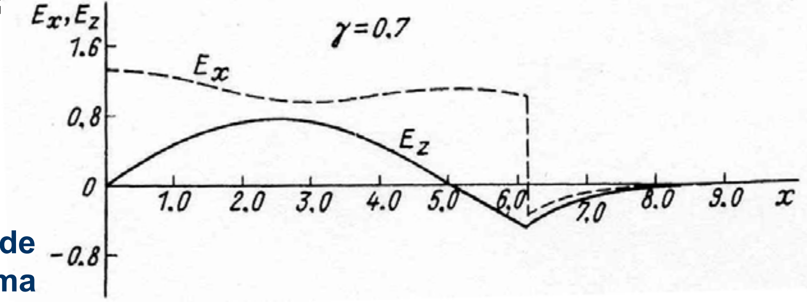
V. B. Gildenburg, 1964
Deformation of plasma
resonance density profile

$q \geq 1$ plasma resonance region

$$\varepsilon(x) = 1 - \frac{\omega_p^2(x)}{\omega^2} = 0$$

$E_x\,|\nabla_x$

Much more complicated
field structure



Field structure in a flat waveguide in a supercritical plasma

Axially symmetric waveguide in above-critical density plasma

A.G. Litvak, G.M. Fraiman

TE case

$$\vec{E} = \vec{\phi}_0 E(\delta)\exp(-i\gamma\kappa_0 z);$$

$$\varepsilon = 1 - q + qE^2; \quad \varepsilon_\infty = 1 - q$$

$$y = (\gamma^2 - \varepsilon_0)^{1/2}\rho; \quad u^2 = E^2 q(\gamma^2 - \varepsilon_0)$$

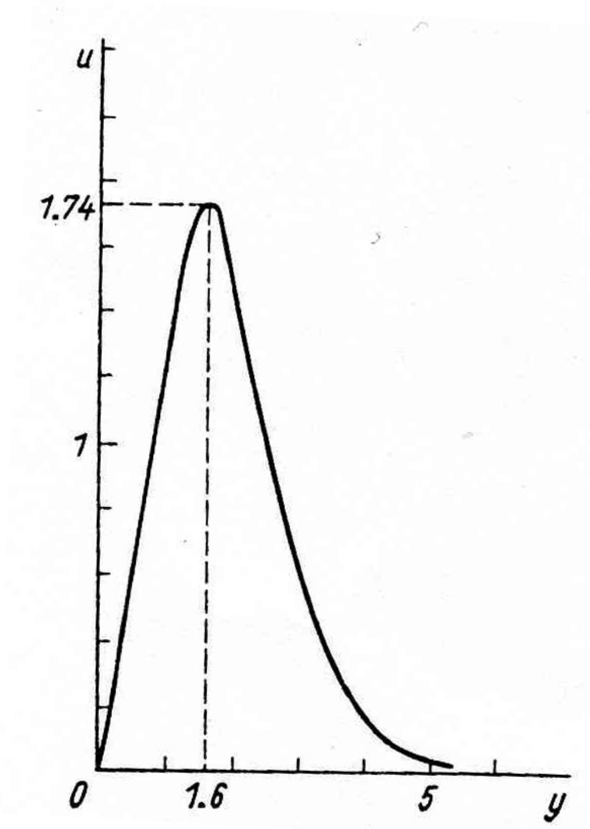
$$\frac{d^2u}{dy^2} + \frac{1}{y}\frac{du}{dy} - u + u^3 - \frac{u}{y^2} = 0$$

$$\gamma^2 = q\left(E_m^2 \big/ E_p^2 u_m^2\right) + \varepsilon_\infty$$

$$P_{total} = \frac{c^3 E_p^2 P^*}{8\omega^2 q}\gamma^2, \quad P^* = 7.69$$

Threshold for self-channeling

$$E_m^2 > 3.04\frac{(q-1)}{q}$$



Field structure
in an axisymmetric waveguide

Mechanisms of plasma nonlinearity

A.G. Litvak, Thesis, 1967; JETP, **30**, 344 (1970) (ZhETP, 1969); Electromagnetic Wave Theory, URSI Symposium, Tbilisi, 1971, pp. 185-191.

$$\varepsilon = 1 - \frac{4\pi e^2 N}{m\omega^2}$$

Weak nonlinearity case
$$\varepsilon = 1 - q - q\underbrace{\frac{\delta N}{N_0}}_{\text{striction, heating, ionization}} + q\underbrace{\frac{\delta m}{m_0}}_{\text{relativistic nonlinearity}}$$

Electrostriction (ponderomotive force)
$$\frac{\partial^2 \delta N}{\partial t^2} - v_s^2 \Delta \delta N = \frac{N_0}{M}\Delta\Phi_e$$

Steady state
$$\delta N = -N_0 \frac{\Phi_e}{T_e + T_i} = -N_0 \frac{|E|^2}{E_p^2}, \quad E_p^2 = 4\left(T_e + T_i\right) m\omega^2 / e^2$$

Heating: weakly ionized plasma
$$\delta N = -N_0 \frac{|E|^2}{E_T^2} \quad E_T^2 = 3Tm\omega^2\, \delta / e^2, \quad \delta \ge \delta_{elastic} = \frac{2m}{M}, \quad L_E \gg l / \sqrt{3\delta}$$

completely ionized plasma (steady state)
$$\delta N = -N\frac{\delta T_e}{2T_0}; \quad l^2 \Delta \delta T_e = -\frac{e^2 |E|^2}{2m\omega^2}, \quad l \text{ - mean free path}, \quad L_E \gg l$$

RELATIVISTIC NONLINEARITY
$$\frac{\delta m}{m} = \frac{|E|^2}{E_{rel}^2}; \quad E_{rel}^2 = 4m^2c^2\omega^2 / 3e^2$$

Application of the general theory of self-focusing to self-action processes in plasma

$$P_{crit} = 5.85\frac{c}{4\pi}\frac{\sqrt{\varepsilon_0}}{\kappa_0^2\varepsilon'} = \frac{5.85}{4\pi}\frac{c^3\sqrt{\varepsilon_0 E_p^2}}{\omega_{p0}^2}$$

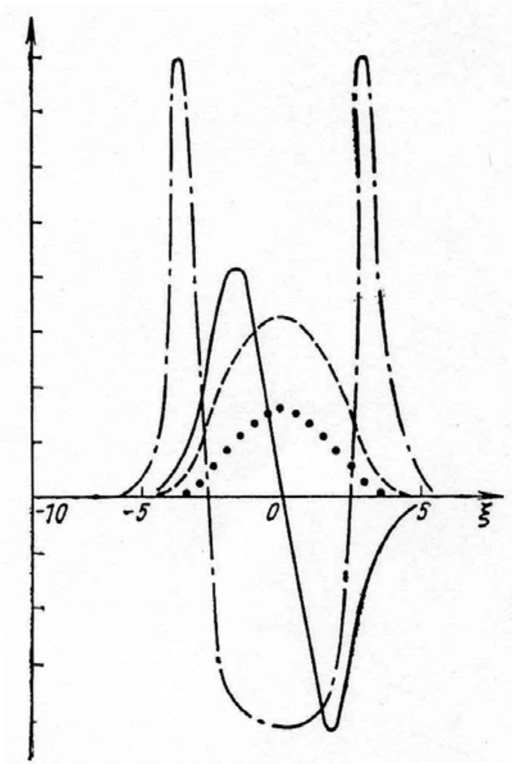
Self-modulation, instabilities, envelope solitons have been investigated.

Relativistically strong solitons in a dense plasma

V.A. Kozlov, A.G. Litvak, E.V. Suvorov, JETP, **49**, 75 (1979)

Major factors: relativistic mass dependence on oscillations energy
relativistic average ponderomotive force
separation of electron and ion charges

Major features: discrete spectrum of solitons
existence of density upper limit
complicated structure compared
with the classical one



Relativistic soliton with marked perturbations of the electron and ion densities for $\omega_{pe}^2/\omega^2 = 2.5$, $\beta = 0.174$, $N_{e\max} \simeq 3.1$, $N_{i\max} \simeq 0.35$. The solid line is the potential profile of the transverse field, the dashed line is the potential profile of the longitudinal field, the dot-dashed line is the electron density profile, and the dotted line is the ion density profile.

Experiments on self-focusing in plasma

1. Thermal self-focusing of a mm-wave beam

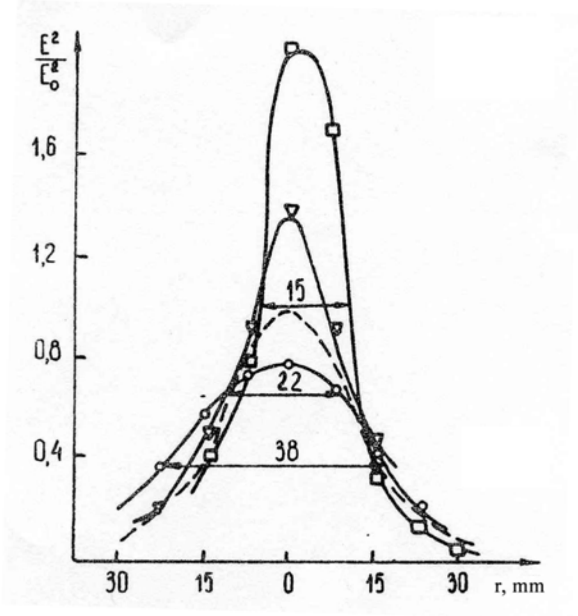
A.G. Eremin, A.G. Litvak, JETP Lett., **13**, 603 (1971)

Gyrotron plasma – new source of HPM in the mm wave band

P = 15 kW, λ = 0.5 cm, τ_{pulse} = 0.4 ms
$N_e = 2{\cdot}10^{13}$ cm^{-3}, $N_{\text{mol}} = 10^{16}$ cm^{-3},
$T_e = 2{\cdot}10^{3\circ}$K, $\nu_{\text{li}} = 2{\cdot}10^{9}$ s^{-1}, $\nu_{\text{cm}} = 2{\cdot}10^{8}$ s^{-1}

$\tau_N >> \tau_{pulse}$ – nonstationary process

Distribution of energy density for *z* = 24 cm:
□ - P_0 = 1 kW,
- P_0 = 0.5 k∇W,
○ - P_0 = 10^{-2} kW



Prediction of the role of the thermal filamentation of radio waves in experiments on ionosphere modification
(on the basis of theoretical evaluations)

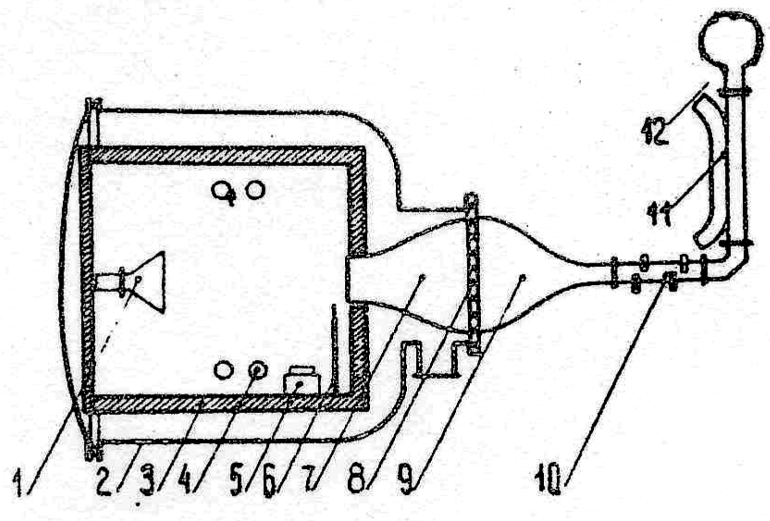
A.G. Litvak, Radiophysica, **14**, 1438 (1968)

Confirmation in the first experiments at Arecibo, where strong foreshortened scattering of the probe wave was observed due to the development of elongated (along the geomagnetic field) density perturbations

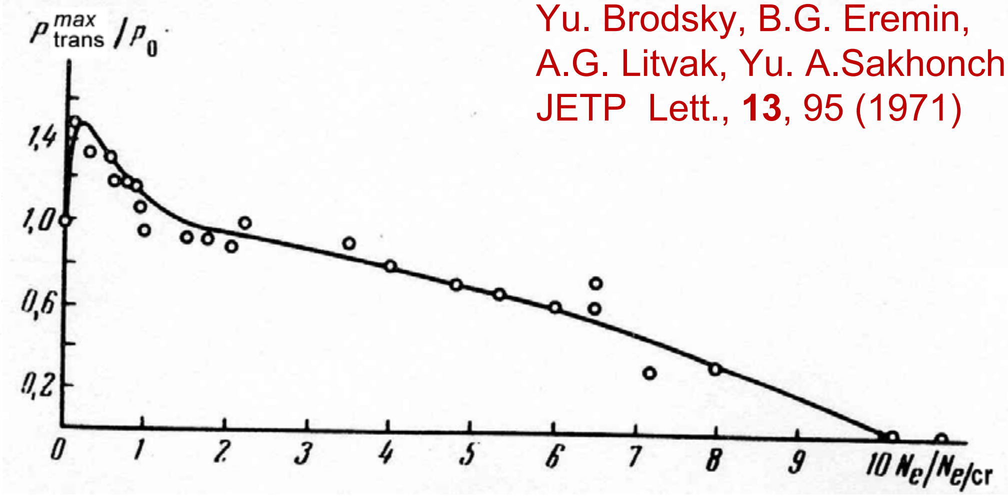
F.W. Perkins, E. Valeo, Phys. PRL, **32**, 1234 (1974)

Nonlinear transparency of above-critical plasma

Yu. Brodsky, B.G. Eremin,
A.G. Litvak, Yu. A.Sakhonchik
JETP Lett., **13**, 95 (1971)



Scheme of the experimental setup: receiving horn (1), vacuum chamber (2), microwave absorber (3), inductor (4), multigrid analyzer (5), Langmuir probe (6), microwave transitions (7 and 9), vacuum sealing (8), matching units (10), directed coupler (11), generator (12)



Maximum power of the signal transmitted through a plasma layer with a thickness of 40 cm as a function of plasma density for the power of the incident pulse P_i = 50 kW, P_0 is the signal power in vacuum, $N_{e\,cr}$ = 10^{11} cm^{-3}, z = 40 cm

P = 300 kW, λ = 10 cm, N_{cr} =10^{11} cm^{-3}, τ_{pulse} = 20 μs

Nonlinear transparency appeared in a time being much shorter than the time of plasma redistribution across the microwave beam.

Theory: splitting of the beam and formation of a narrow channel due to modulational instability.

Relativistic Nonlinear Optics

RNO has become essentially significant due to the progress in the development of TW-PW laser sources and their possible applications:

1. Laser-plasma particle acceleration

Beat-wave excitation* of plasma wave

Wake-field excitation – major line after CPA development

T. Tajima, J. Dawson,
“Beat electromagnetic waves excitation of plasma waves”,
PRL, **43**, 267 (1979)

* A.G. Litvak, *On nonlinear excitation of plasma waves,* Radiophysica, **7**, 562 (1964)

2. Fast ignition of fusion pellet

M. Tabak et al.,
Phys. Plasma, **1**,
1626 (1994)

Idea to use relativistic waveguiding of ultrashort laser pulses to increase the length of plasma wake-field excitation for particle acceleration

L.A. Abramyan, A.G. Litvak, V.A. Mironov, A.M. Sergeev, JETP, **102/1816** (1992)

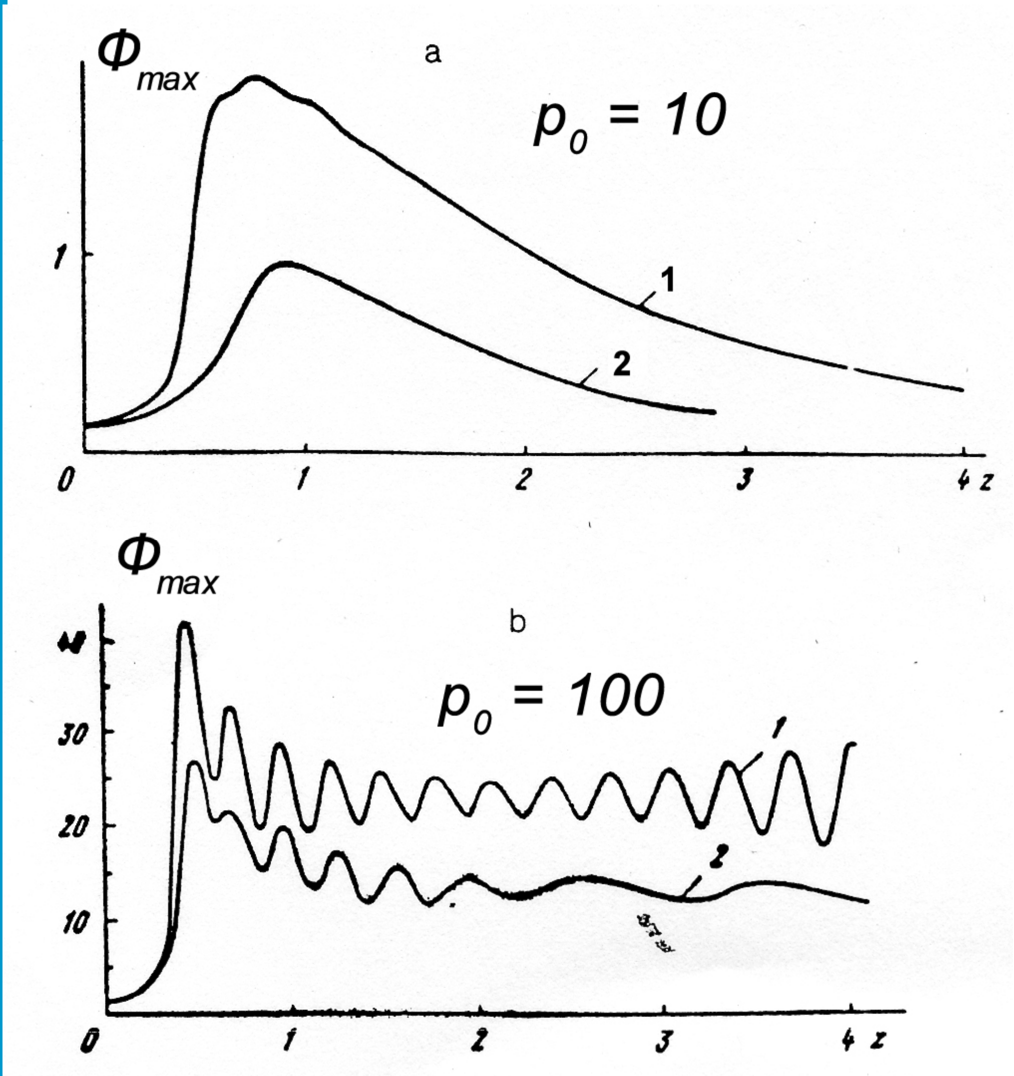
$$-2ik\frac{\partial A}{\partial z}=\frac{\omega_p^2}{u^2\omega^2}\frac{\partial^2 A}{\partial\tau^2}+\Delta_\perp A+\frac{\omega_p^2}{c^2}\frac{\Phi}{1+\Phi}A=0$$

$$\frac{\partial^2\Phi}{\partial\tau^2}=\omega_p^2\frac{1+|A|^2-(1+\Phi)^2}{2(1+\Phi)^2},$$

$$\Phi=\frac{\phi}{m_0c^2},\quad A=\frac{e\tilde{A}}{mc^2},\quad \tau=t-z/u,\quad u$$ **- group velocity**

Self-similar solutions for weak nonlinearity

Paraxial optics with nonlinearity saturation (computations)



Dependence of plasma wave potential on axis *z* for the Gaussian pulse:
1 – aberration-free approximation,
2 – computations

Self-action of a Few-Cycle Pulse

Allowance for the dependence of the group velocity on intensity $V_{gr}(E)$

A.A. Balakin et al.,
Phys. Rev. A78,
061803 (2008)

Propagation of a wide-band circularly polarized wave packet

along z without reflection $E\left(z,\tau=t-z,\vec{r}_{\perp}\right)$

$$\frac{\partial}{\partial\tau}\left(\frac{\partial E}{\partial z}+3E^{2}\frac{\partial E}{\partial\tau}-b\frac{\partial^{3}E}{\partial\tau^{2}}-c\frac{\partial^{2}E}{\partial\tau^{2}}\right)+aE=\Delta_{\perp}E$$

$$E=\Phi'_{\tau},\quad I=\int\Phi_{\tau}^{2}\,d\tau d\vec{r}_{\perp}=const$$

Dispersion $k_{z}=-{}^{a}\!/\!_{\omega}+b\omega^{3}$

$$\text{Hamiltonian}\;\; H=\int\left\{\left|\nabla_{\perp}\Phi\right|^{2}-b\left|\Phi_{\tau}\tau\right|^{2}-\frac{1}{2}\left|\Phi_{\tau}\right|^{2}+a\left|\Phi\right|^{2}\right\}d\tau d\vec{r}_{\perp}=const$$

$$a_{eff}^{2}=\frac{\int r_{\perp}^{2}\left|\Phi_{\tau}\right|^{2}d\tau d\vec{r}_{\perp}}{I}\qquad\frac{d^{2}a_{eff}^{2}}{d^{2}z^{2}}=\frac{8H+8\int\left(b\left|\Phi_{\tau\tau}\right|^{2}-a\left|\Phi\right|^{2}d\tau d\vec{r}_{\perp}\right)}{I}$$

Dispersion-free medium $a=b=0$ $\qquad H<0$ - collapse

New type of collapse where wave breaking near the rear front of the pulse leaves the collapse of the wave field somewhat behind:

$$z_b < z_f$$

Analytical results and computations for the dispersion-free case and initial

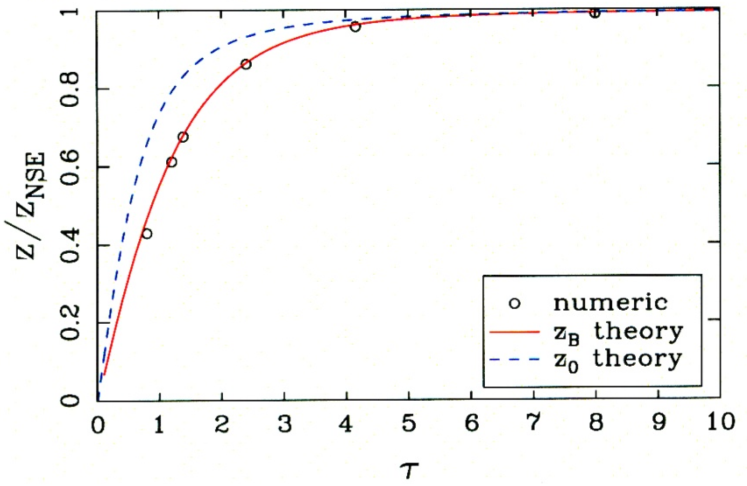
$$\Phi = A\cosh^{-1}\left(\tau/\tau_p\right)\exp\left(-\frac{r_\perp^2}{2\rho_0^2}\right)$$

$$H = 2\pi A^2\tau_p\left(1 - I/I_{cr}\right),$$

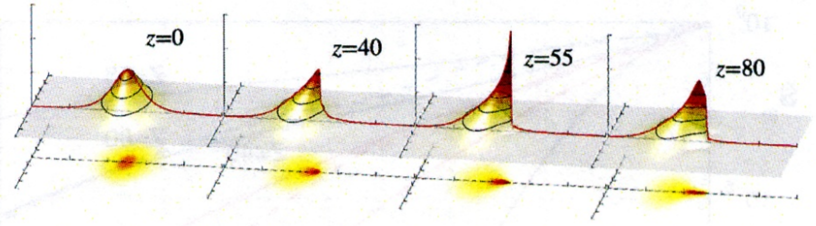
$$I_{cr} = 140\pi\tau_p^3\left(1+3k^2\right)/\left(35k^4+14k^2+3\right)$$

$$I = 2\pi A^2\rho_0^2\left(1+3k^2\right)/3\tau_p\,, \qquad k = \omega\tau_p$$

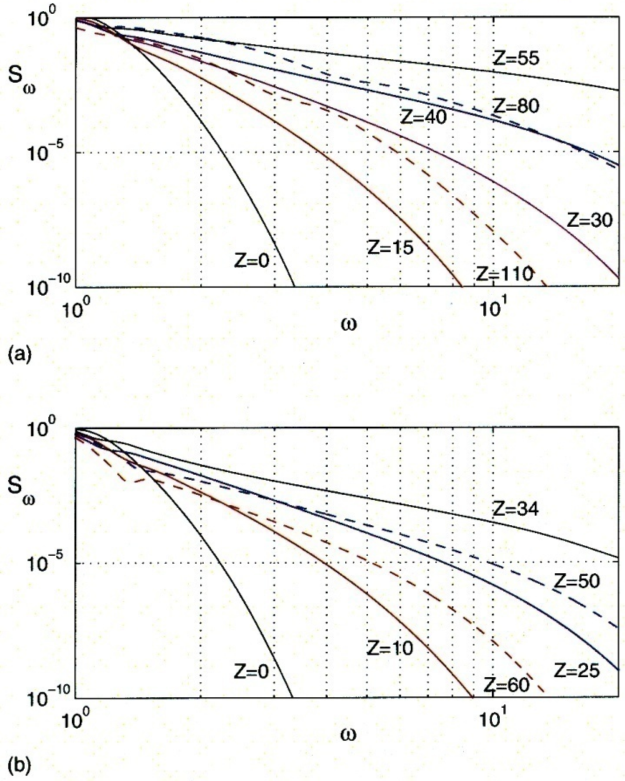
$$z_f = \frac{\rho_0^2}{2\sqrt{3\tau_p}\sqrt{\left(1+3k^2\right)\cdot\left(I/I_{cr}-1\right)}}$$



Length of self-focusing z_0 (blue dashed line);
the red line is the overturn distance.
○ is the length of the singularity obtained
by numerical simulation for $a = b = 0$.



Dynamics of the circularly polarized field
$|u(z, \tau, r)|$ in a dispersion-free medium ($a = b = 0$)
for $\gamma = 0.04$. Distribution of the field at the
boundary of the nonlinear medium $u = 0.6$
$e^{i\tau}/\cosh(0.3\tau)\exp(-r^2/100)$.



Short-wave part of the spectrum
S (ω>1) at the beam axis:
(a) for $a = 0$ and $b = 0$;
(b) for $a = 1$ and $b = 0$

Self-compression of relativistic strong femtosecond pulses during wake-field excitation

A.A. Balakin, A.G. Litvak,
V.A. Mironov, S.A. Skobelev,
PRL (to be published)

Reflection-free approximation for wide-band packet in transparent plasma $\omega_p << \omega_0$

$$\frac{\partial^2 A}{\partial z \partial \dot{\tau}} + \frac{\Phi \beta A}{1+\Phi} = \Delta_{\perp} A$$

$$\frac{\partial^2 \; \Phi}{\partial \tau^2} = \frac{\omega_p^2}{2\omega_0^2}\left[\frac{1+A^2}{\left(1+\Phi\right)^2} - 1\right].$$

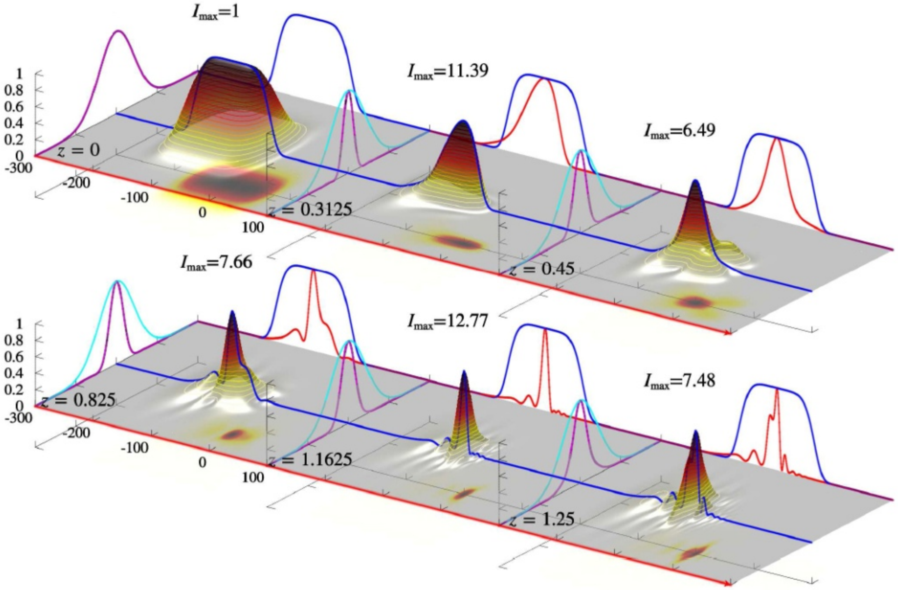
Let us consider $\tau_{\text{pulse}} < \lambda_{\text{plasma}} = \dfrac{2\pi c}{\omega_p}$; laser pulse radius $\rho_0 > {}^{c}\!/_{\omega_p}$.

No bubble formation: transverse transit time of electrons $\tau_{\perp} = {}^{\rho_0}\!/_{c} \gg \tau_{\text{pulse}}$

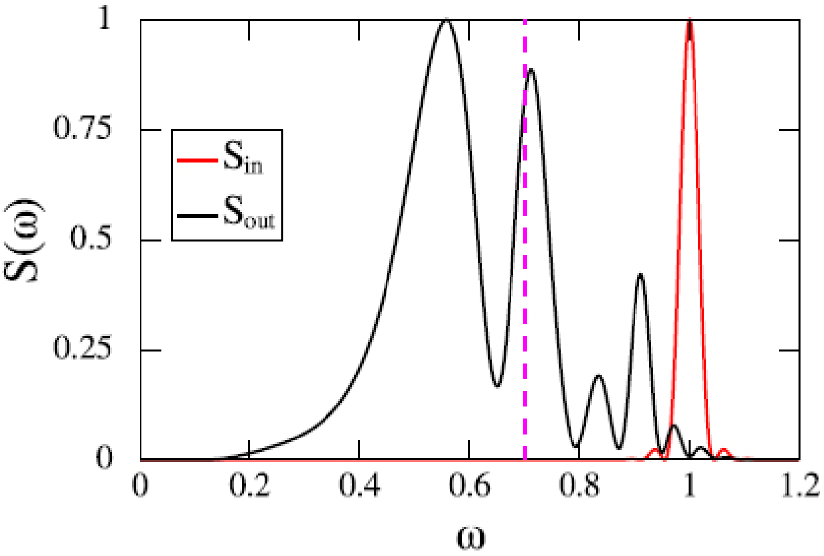
Principal role of self-focusing and nonlinearity saturation

Computations for the super-Gaussian pulse $A = A_0 \exp\left(-\ln\left(2\, {}^{r^2}\!/_{2}\right) + i\tau - 32\ln 2 \cdot \left({}^{\tau}\!/_{\tau_p} \right)^6 \right)$

Pulse P = 1 PW; τ_p = 53 fs (20 cycles); λ = 0.8 μm; r_0 = 120 μm; $Ne = 7 \cdot 10^{17}$ cm^{-3} could be compressed to τp = 7 fs.



Dynamics of the laser pulse intensity *I* for τ_p = 40 π, a_0 = 2. Here, the blue line is the initial time profile of the laser pulse along the beam axis; the red line is the current time profile of the pulse along the beam axis; the cyan line is the initial distribution of the field intensity in the transverse direction; and the magenta line is the current distribution of the laser pulse in the transverse direction.



Field spectrum of the laser pulse. Self-compression of the pulse is almost uniform over the cross section.