SPACE – TIME ANALOGIES IN NONLINEAR OPTICS

(personal experience in 1965-2011)

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Spatial self-focusing

$$\frac{\partial A}{\partial z} + iD\Delta_{\perp} A + i\gamma |A|^2 A = 0 \qquad D = 1/2k;\, \gamma = k_0 n_2 / n_0$$

Aberration free method (Akhmanov, Sukhorukov, Khokhlov, 1966)

$$A = \frac{E}{a_x(z) a_y(z)} \exp\left(-\frac{x^2}{a_x^2(z)} - \frac{y^2}{a_y^2(z)} - i\frac{k x^2}{2 a_x(z)}\frac{da_x}{dz} - i\frac{k y^2}{2 a_y(z)}\frac{da_y}{dz}\right)$$

$$\frac{d^2 a_x}{dz^2} = \frac{l_{dif,x}^2}{a_x^3} - \frac{l_{nl}^2}{a_x^2 a_y} \qquad \frac{d^2 a_y}{dz^2} = \frac{l_{dif,y}^2}{a_y^3} - \frac{l_{nl}^2}{a_y^2 a_x}$$

$$a_x = a_y = a_0 (1 - z^2 / l_{sf}^2)^{1/2},\, l_{sf} = l_{dif} (P_0 / P_{cr} - 1)^{1/2};\, P_{cr} = \frac{c\lambda^2}{256 n_2}$$

Hydrodinamic Analogy

Self-defocusing is like superfluidity .

$$A = B\exp(-kS) \qquad \frac{\partial S}{\partial z} + \frac{1}{2}\left(\frac{\partial S}{\partial x}\right)^2 = n_2 B^2 \qquad \theta = \frac{\partial S}{\partial x}$$

$$\frac{\partial \theta}{\partial z} + \theta \frac{\partial \theta}{\partial x} = n_2 \frac{\partial B^2}{\partial x} \qquad \frac{\partial B^2}{\partial z} + \frac{\partial\left(\theta B^2\right)}{\partial x} = 0$$

Fluid of superfluid liquid (Gross-Pitaevskii equation) is similar to defocusing of an optical beam.

(Akhmanov, Sukhorukov, Khokhlov, 1968)

Eikonal equation to describe the caustic, for example, the outer ring with defocusing. From the viewpoint of hydrodynamic the caustic formation means shock wave is formed. Therefore, a shock wave in a Bose condensate can be studied in nonlinear optics as an example of defocusing in a photorefractive crystal.

(Kamchatnov at all, 2005)

Space-time analogy for optical beams and pulses

$$\frac{\partial A}{\partial z} + \beta \frac{\partial A}{\partial x} + i D \frac{\partial^2 A}{\partial x^2} = i k n_{nl} \left(|A|^2 \right) A \qquad \frac{\partial A}{\partial z} + \nu \frac{\partial A}{\partial \tau} + i D \frac{\partial^2 A}{\partial \tau^2} = i k n_{nl} \left(|A|^2 \right) A$$

- Beam $\longrightarrow$ Pulse
- Diffraction $\longrightarrow$ Dispersion

$$x \longrightarrow \tau = t - z/u$$

$$D_j = \frac{1}{2k_j} \longrightarrow D_j = -\frac{1}{2}\frac{\partial^2 k_j}{\partial \omega^2}$$

$$\beta \longrightarrow \nu = 1/u_1 - 1/u_j$$

Temporal self-action

$$\frac{\partial A}{\partial z} + iD\frac{\partial^2 A}{\partial \tau^2} = -i\gamma |A|^2 A \qquad D = -\frac{1}{2}\frac{\partial^2 k}{\partial \omega^2};\ \gamma = k_0 n_2 / n_0$$

$$n_2 D = -\frac{n_2}{2}\frac{\partial^2 k}{\partial \omega^2} < 0$$

Decompression

$$n_2 D = -\frac{n_2}{2}\frac{\partial^2 k}{\partial \omega^2} > 0$$

Self-compression

$$A = \frac{E}{T(z)}\exp\left(-\frac{\tau^2}{T^2(z)} - i\frac{\tau^2}{4DT}\frac{dT}{dz}\right)$$

$$\frac{d^2 T}{dz^2} = \frac{l_{diis}^2}{T_x^3} - \frac{l_{nl}^2}{T^2} \qquad k\, n_2 E_{sol}^2 T^2 = -4D$$

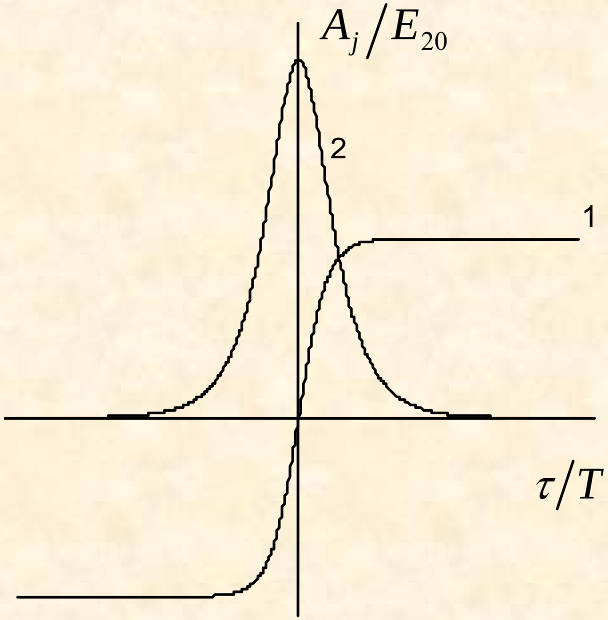
Two-wave interaction

Giant parametric pulse. Amplification in two-level medium.

$$\frac{\partial A_1}{\partial z} + \frac{1}{u_1}\frac{\partial A_1}{\partial t} = -\gamma A_2 A_1$$

$$\frac{\partial A_2}{\partial z} + \frac{1}{u_2}\frac{\partial A_2}{\partial t} = \gamma A_1^2$$

$$u_1 \neq u_2$$



$$E_{s2} = E_{02} th(\tau_s / T_s);$$

$$E_{s1} = E_{01} ch^{-1}(\tau_s / T_s)$$

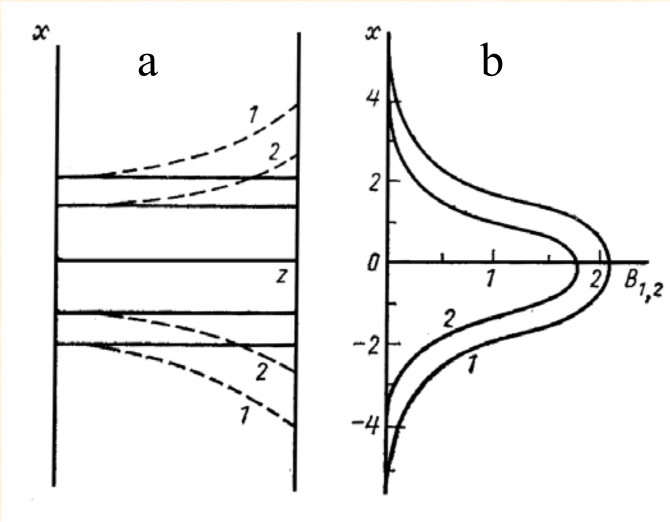
$$E_{01} >> E_{02};$$

$$u_{s1} > u_1$$

Interaction of FH kink – dark soliton (1) and SH giant bright soliton (2) in the parametric traveling wave amplifier with group velocity mismatch. Akhmanov, Sukhorukov, Khokhlov, Chirkin, Kovrigin. 1968.

FH first harmonic of the inverted medium. The signal runs at a different velocity and at the forefront of licks pump energy. As a result, the signal amplitude can exceed the pump amplitude dozens of times. This was confirmed experimentally by Piskarskas, 1977.

Parametric (quadratic) soliton



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$$\omega_3 = \omega_1 + \omega_2 \qquad k_3 = k_1 + k_2 + \Delta k$$

$$\frac{\partial A_1}{\partial z} + iD_1\Delta_\perp A_1 + i\gamma A_2^* A_3 e^{-i\Delta kz} = 0$$

$$\frac{\partial A_2}{\partial z} + iD_2\Delta_\perp A_2 + i\gamma A_2^* A_3 e^{-i\Delta kz} = 0$$

$$\frac{\partial A_3}{\partial z} + iD_3\Delta_\perp A_3 + i\gamma A_1 A_2 e^{i\Delta kz} = 0$$

Soliton propagation of beams in quadratic medium: a -schematic representation of coupled beams. . Dashed lines show the diffraction divergence of Gaussian beams in a linear medium; *b* – envelope of 2D soliton.

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$$A_j(x) = E_j ch^{-2}(x/a)e^{-iq_j z} \qquad q_1 = l_{dif}^{-1} = 2/k_1 a^2;\, q_2 - 2q_1 + \Delta k = 0$$

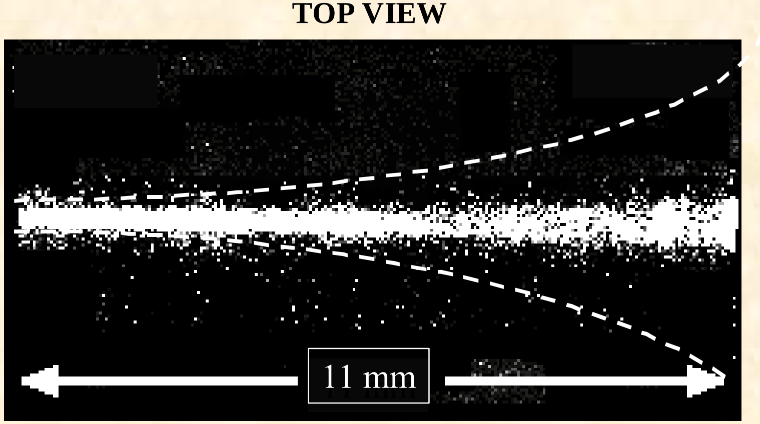
$$D_1 = 2D_2;\, E_2 = \sqrt{2}E_1;\, \gamma_1 E_1 = 3\sqrt{2}D_1 a^{-2}$$

Quadratic soliton envelope is a hyperbolic secant squared

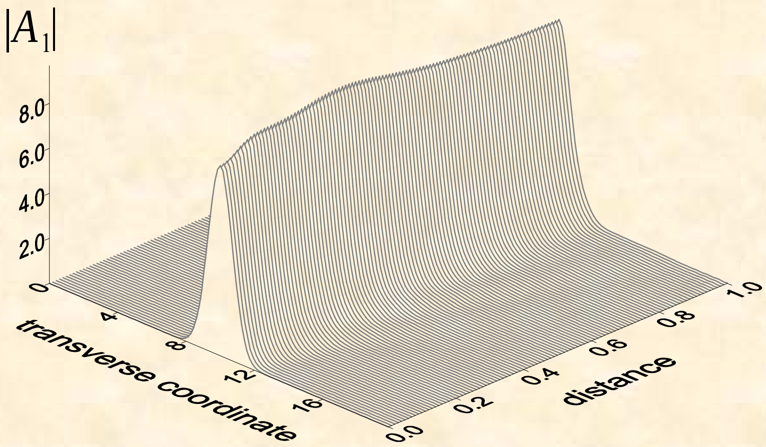
Karamzin, Sukhorukov 1974

Experiments on Quadratic Solitons

TOP VIEW



5 Diffraction Lengths



Photograph of a 20 μm wide quadratic soliton propagating over 5 diffraction lengths in bulk KNbO3 at 982 nm forType I non-critical phase-matching.

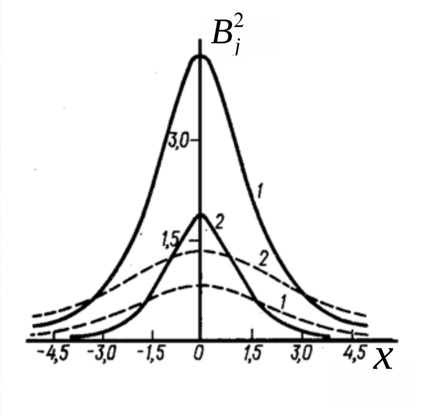
Numerical simulation of soliton traping

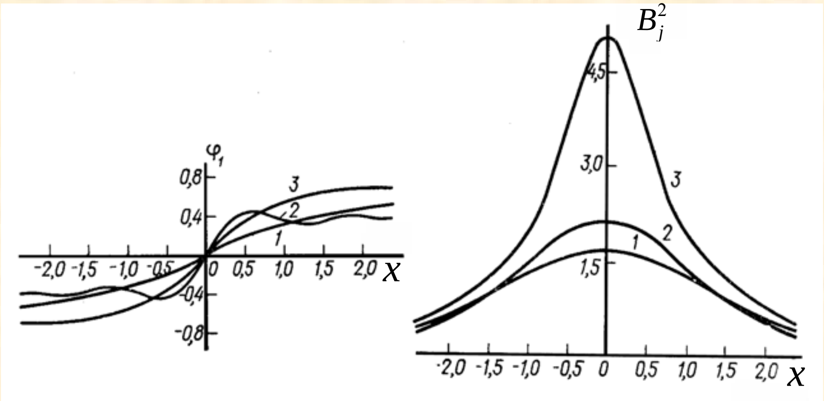
GEORGE I. STEGEMAN 1995

$q_1 / D_1 = 1$

Temporal parametric (quadratic) soliton..

Balance between dispersion and nonlinearity,





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Numerical simulations of parametric solitons envelopes under group-velocity matching and

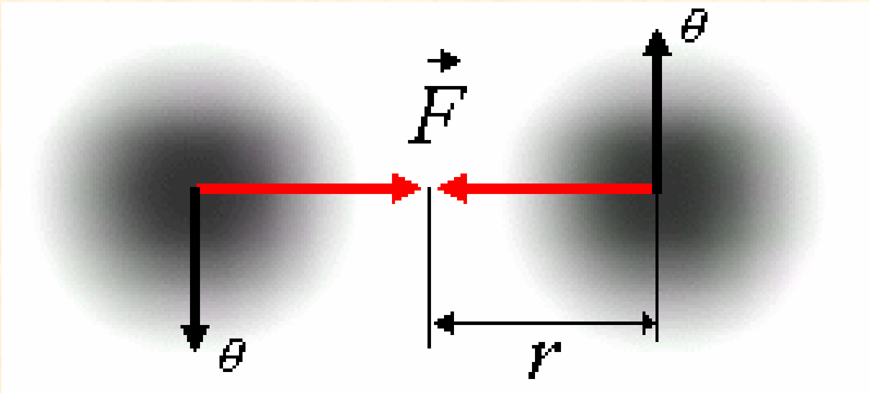
$q_1 / D_1 = 1 \qquad q_2 / D_2 = 9$

Results of numerical calculations for soliton phase and amplitude profiles with

$q_j = D_j \quad \Delta q_x a = 1\ (1),\ 2\ (2),\ 3\ (3)$

$$\frac{\partial A_1}{\partial z} + iD_1 \Delta_\perp A_1 + i\gamma A_2^* A_3 e^{-i\Delta kz} = 0 \quad D_j = -\frac{1}{2}\frac{\partial^2 k_j}{\partial \omega^2}; \nu_{1j} = u_1^{-1} - u_j^{-1}$$

Analogy between quasi-particle interaction and beam (soliton) interactio



coordinate of center of solitons

$$x_0 = r\cos\varphi$$

$$y_0 = r\sin\varphi$$

Λ

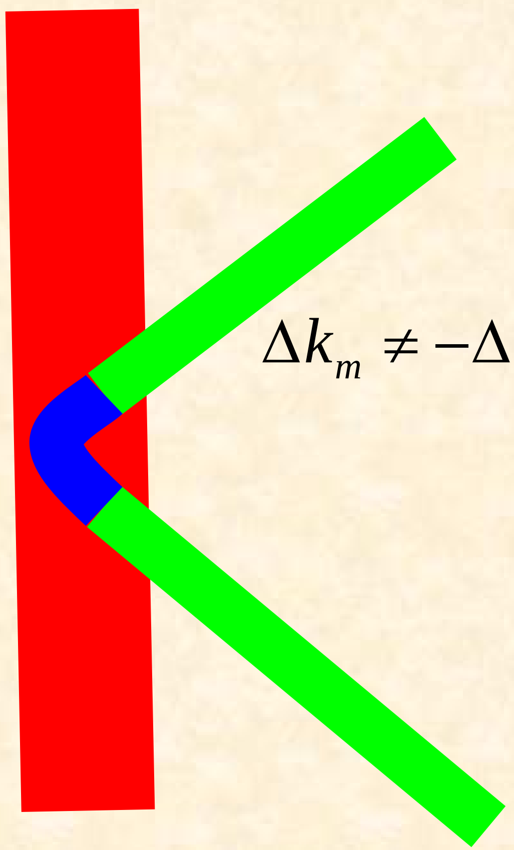
$$m_s \frac{d^2 y_0}{d z^2} = -F y_0 / r \qquad m_s \frac{d^2 x_0}{d z^2} = -F x_0 / r$$

The period of soliton spiral can be found by equilibrium angle and spiral radius

$$\Lambda = 2\pi r / \theta_0$$

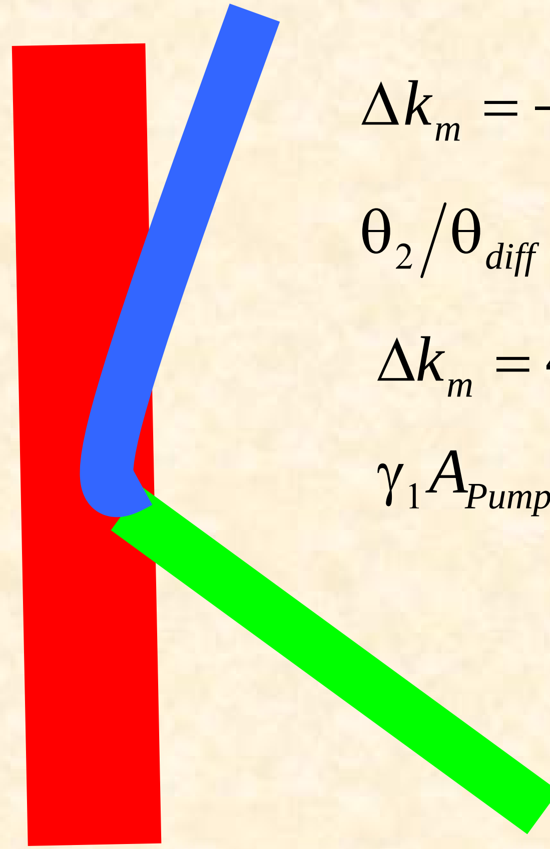
COMPARISON OF PHASE MISMATCHED AND PHASE-MATCHED PARAMETRIC REFLACTIONS

- **Mismatched reflection of the signal wave**



$$\Delta k_m \neq -\Delta k_t$$

- **Phase-matched reflection with up-conversion**



$$\Delta k_m = -\Delta k_t$$

$$\theta_2 / \theta_{diff} = 24$$

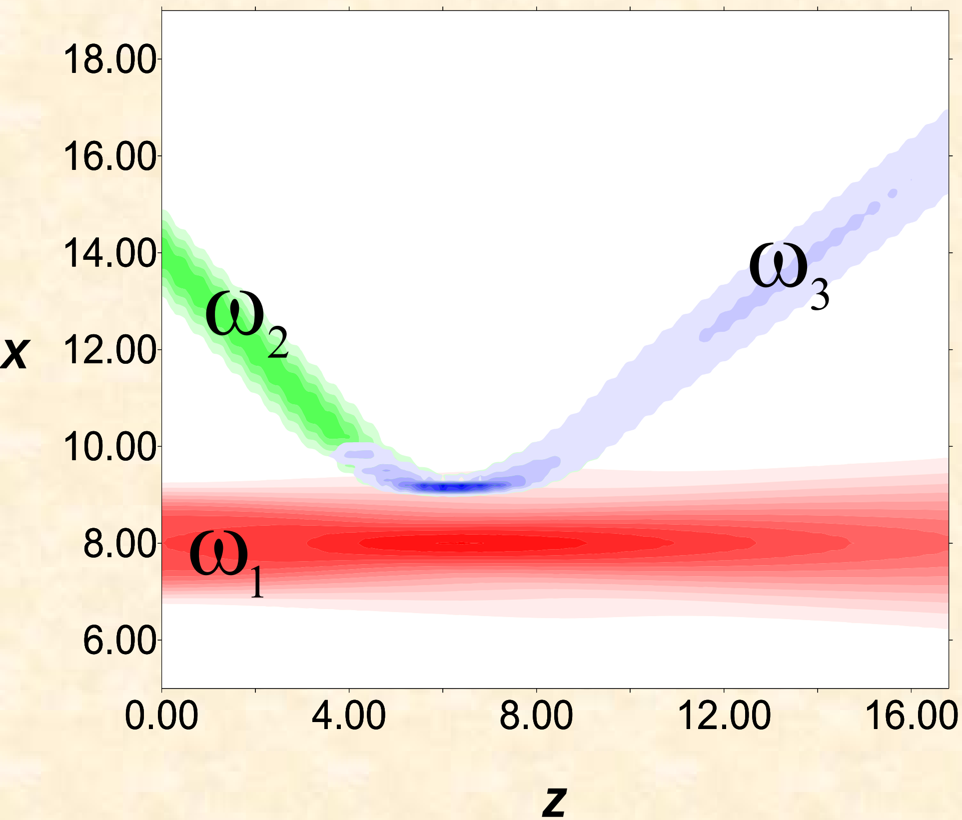
$$\Delta k_m = 40/6$$

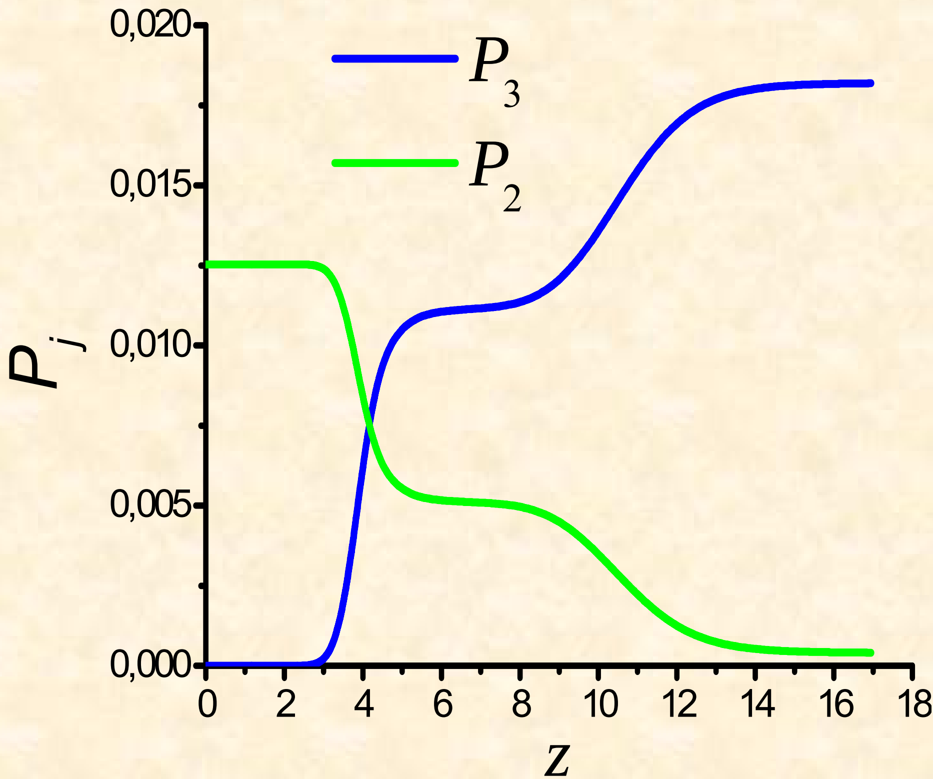
$$\gamma_1 A_{Pump} = 15$$

PHASE-MATCHED PARAMETRIC REFLECTION

$$\Delta k = 0$$

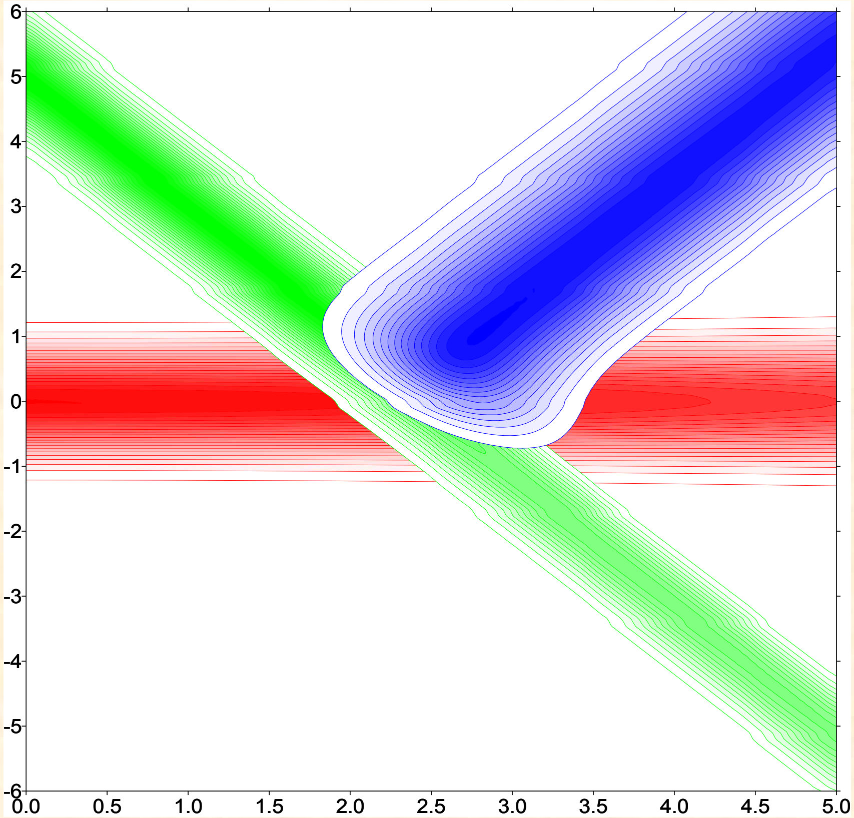
- **Phase-matched parametric reflection with frequency conversion**
- Energy exchange

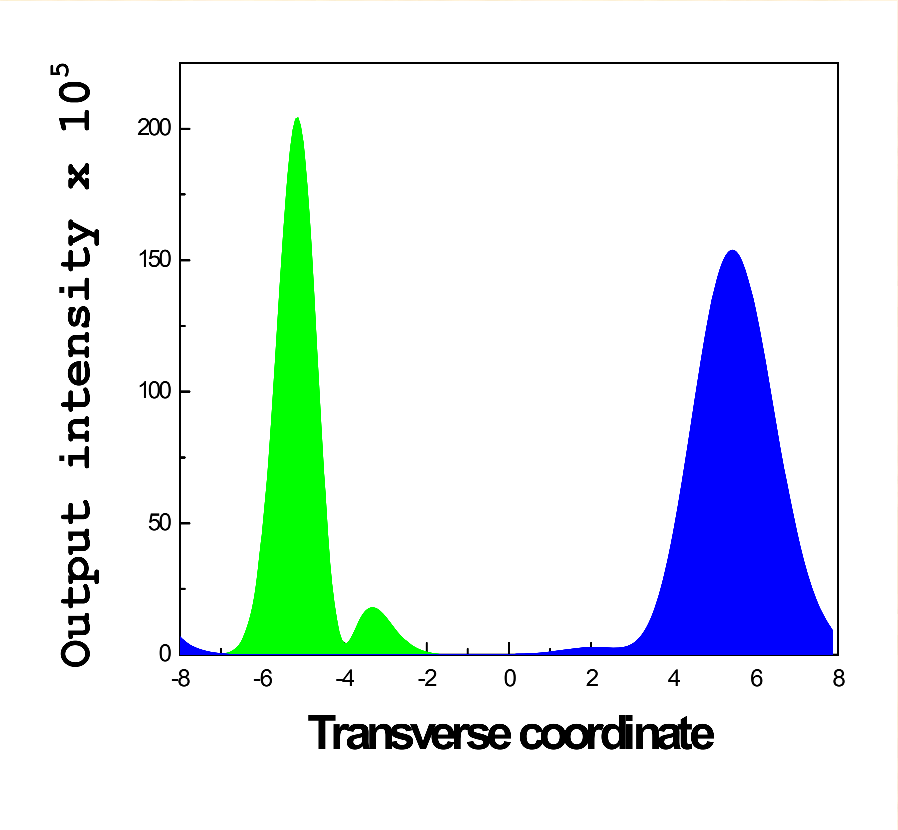




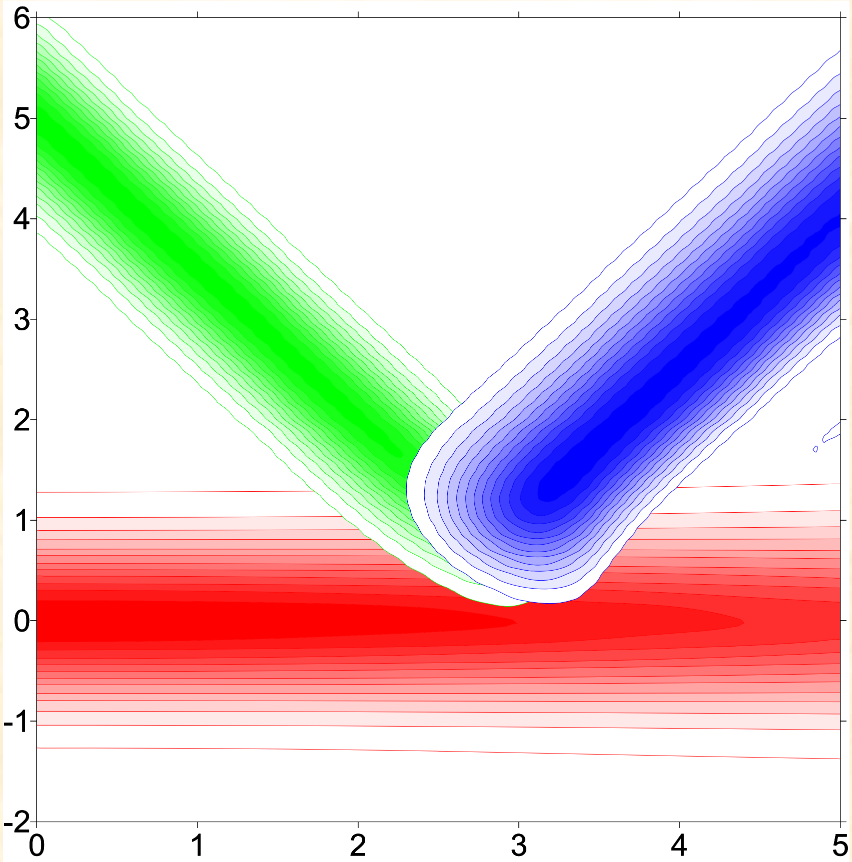
Partial reflection with weak pump beam

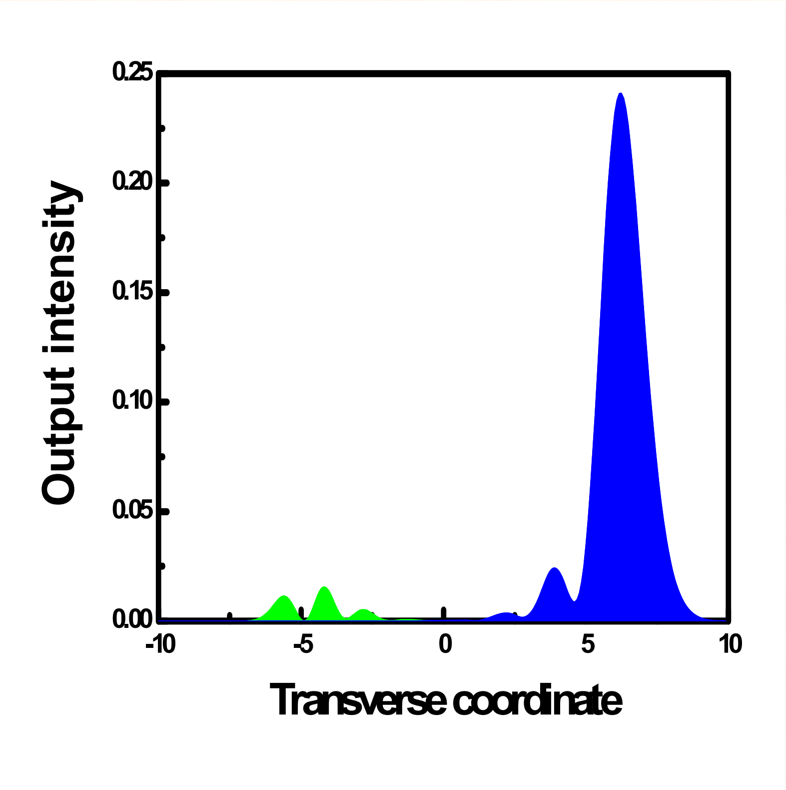
$$G_m = 0.77$$



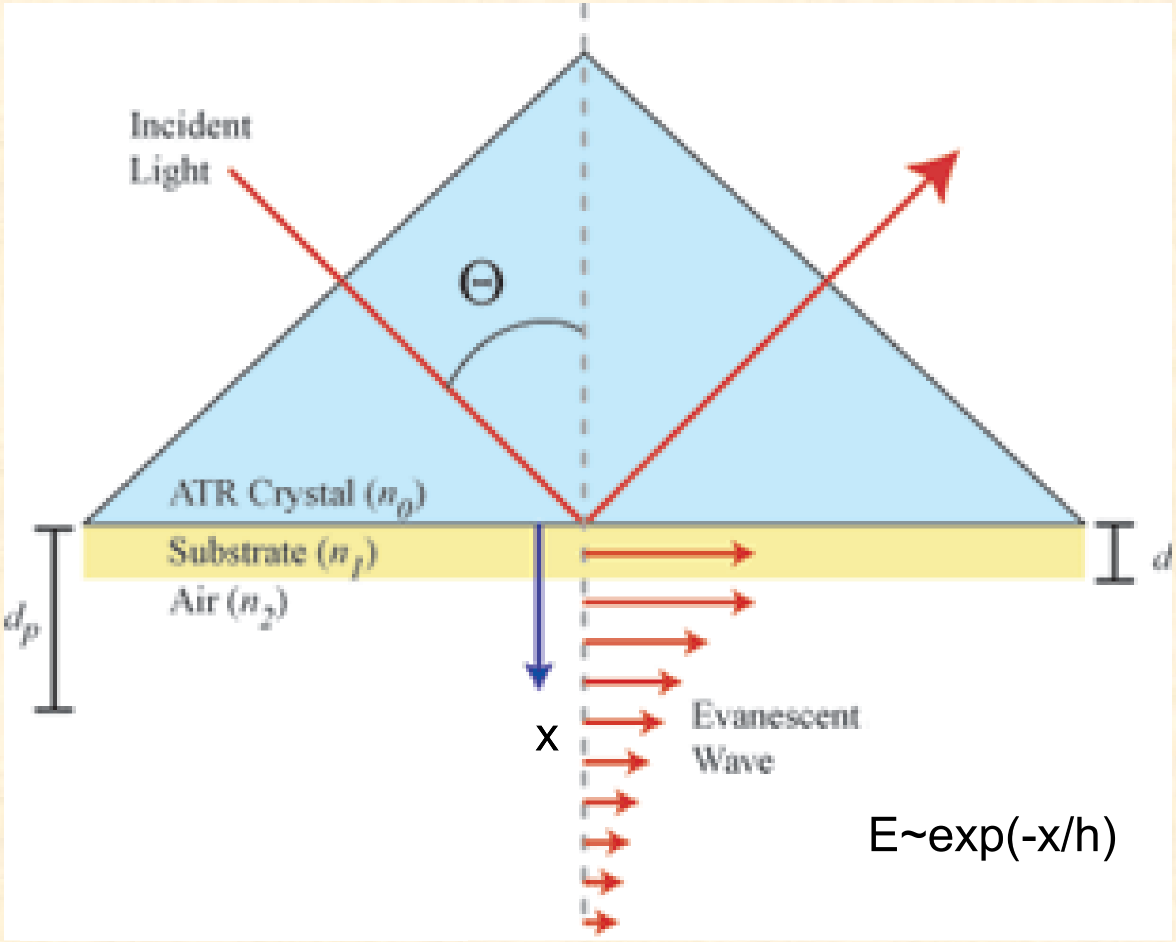


Parametric reflection with intensive pump beam $G_h = 1.94$



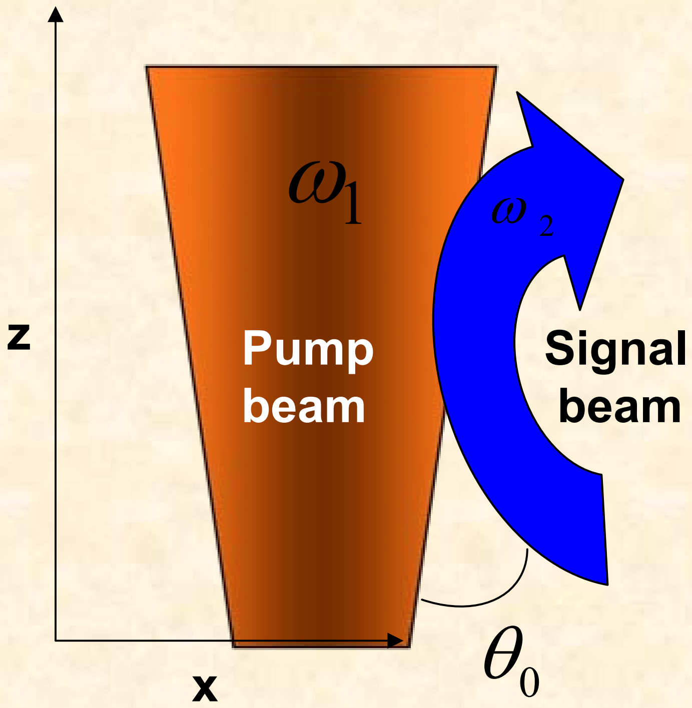


Total internal reflection in a prizm



$$\sin \theta_C = \frac{n_2}{n_1} \sin 90^\circ = \frac{n_2}{n_1}$$

REFLECTION OF OPTICAL BEAM FROM THE LASER-INDUCED INHOMOGENEITY



The trajectory equation in a pump field looks like:

$$\frac{dx}{dz} = \pm\sqrt{2}\sqrt{\theta_0^2/2 + n_{nl}(x) - n_{nl}(x_0)}$$

$$n = n_0 + n_{nl}(x)$$

In a point of turn dx/dz = 0, and

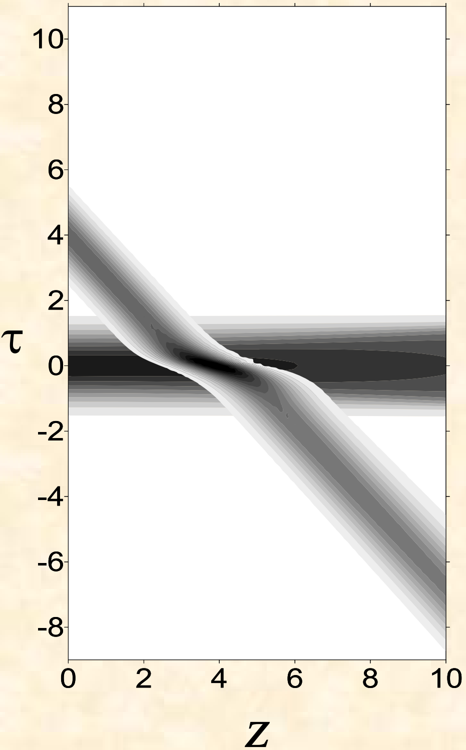
$$n_{nl}(x_{turn}) = -\theta_0^2/2 + n_{nl}(x_0) < 0$$

$\longrightarrow$ **Nonlinear reflection: θ_0 ➡ – θ_0**

Total reflection ocurs with crossing angles smaller than critical value:

$$\theta \leq \theta_{cr} \qquad \theta_{cr} = \sqrt{2|n_{nl}(0)|}$$

Collision of two pulses in the nonlinear dispersive medium (wave theory)

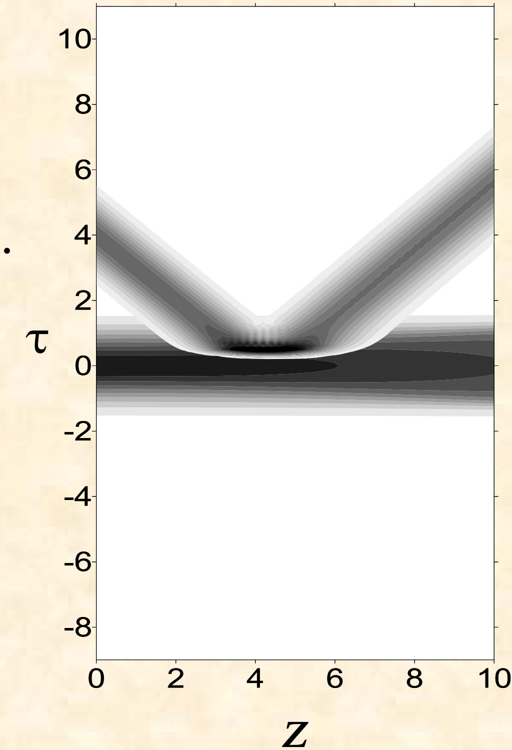


$$\frac{\partial A_1}{\partial z} + i\,D_1 \frac{\partial^2 A_1}{\partial \tau^2} = -i\,k_1\,n_{nl} \left|A_1\right|^2 A_1.$$

$$\frac{\partial A_2}{\partial z} + \nu_2 \frac{\partial A_2}{\partial \tau} + i\,D_2 \frac{\partial^2 A_2}{\partial \tau^2} = 2ik_2 n_{nl} \left|A_1\right|^2 A_2.$$

$$A_j(z=0) = E_j \exp\left(-(\tau-\theta_j)^2 / T_j^2\right)$$

$$E_1 >> E_2$$

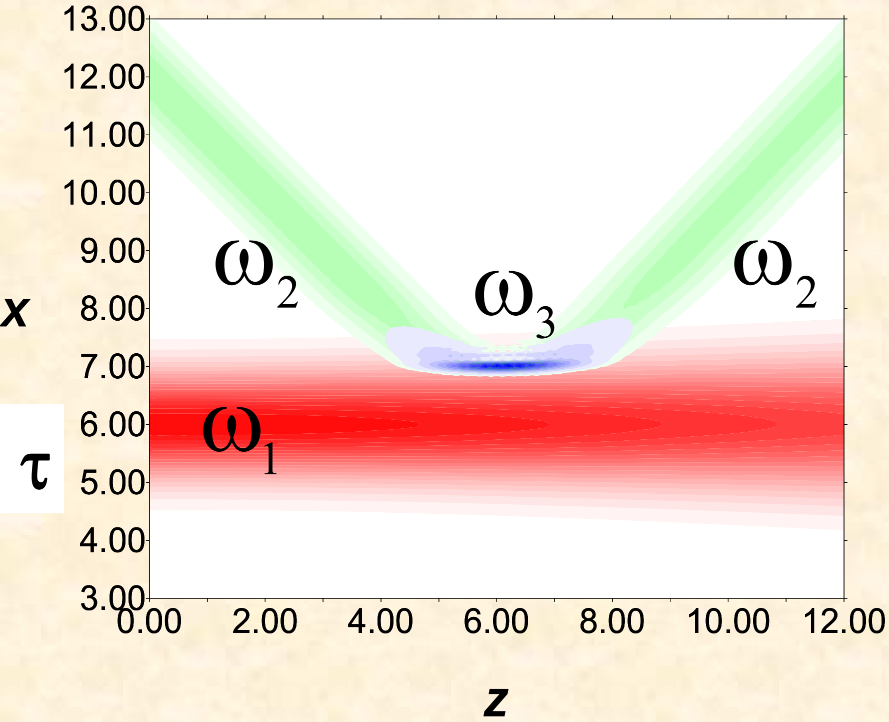


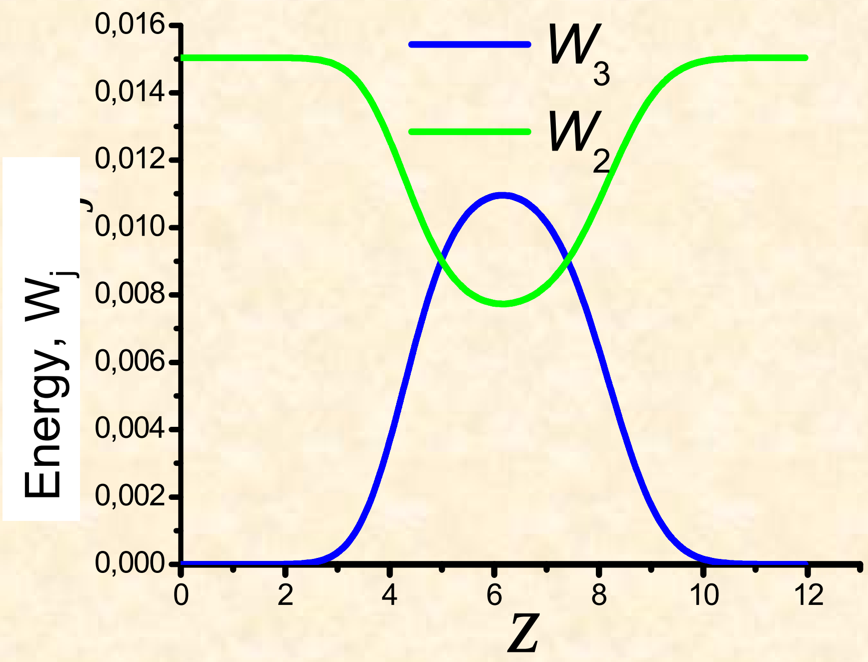
Pattern of two-pulse collision on the plane (*τ, z*): (left) small amplitude of the basic (pump) beam; (right) total reflection of signal pulse from basic pulse

Parametric reflection with the cascade interaction

Displays basic (red), signal (green) and idler (blue) pulses in space-time domain

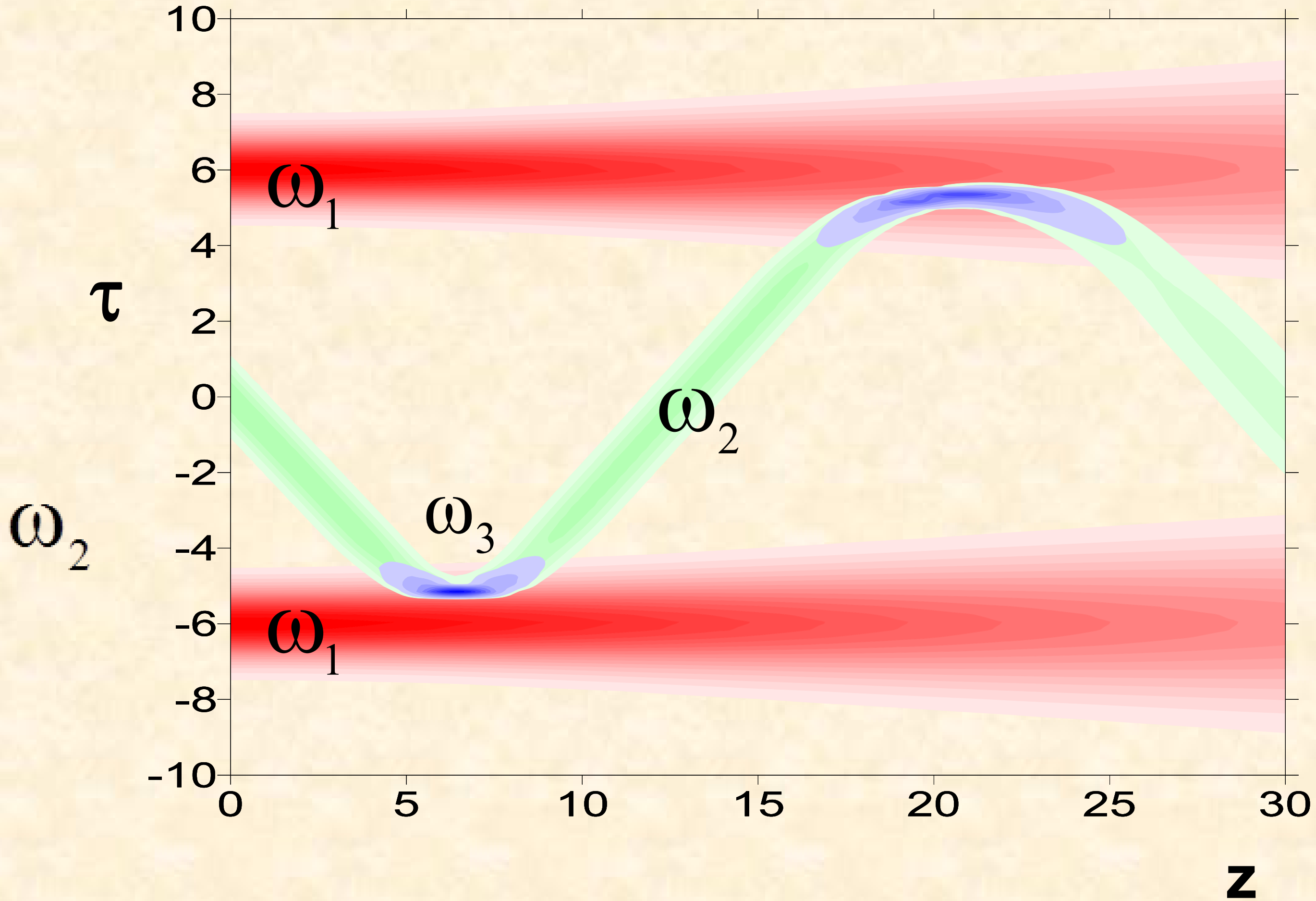
- The energy exchange between the pulses



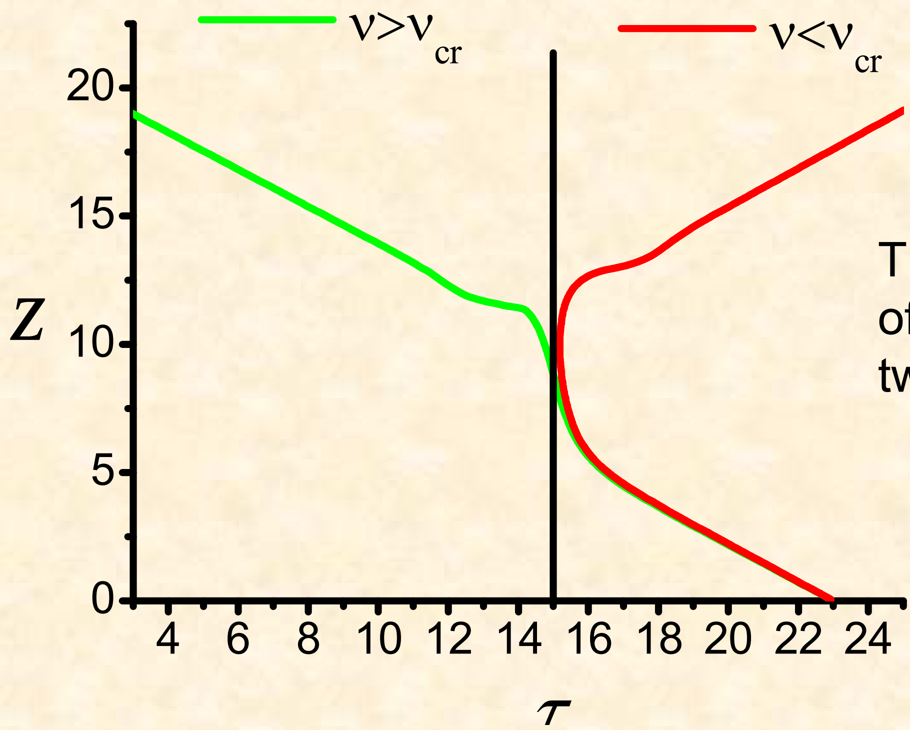


Trapping signal pulse by two pump pulses

The signal pulse (green) is traped by two pump pulses (red), alternately reflecting from them . It is seen that the reflection decreases due to the dispersion broadening of the pump pulses.



The dynamics of the signal pulse collision at small and large group-velocity mismatch

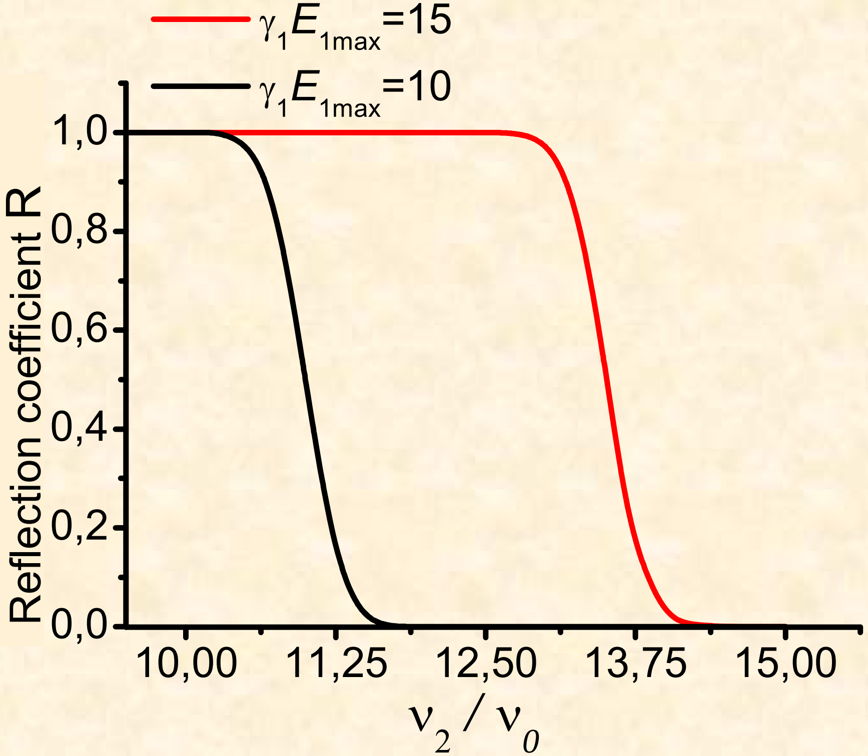


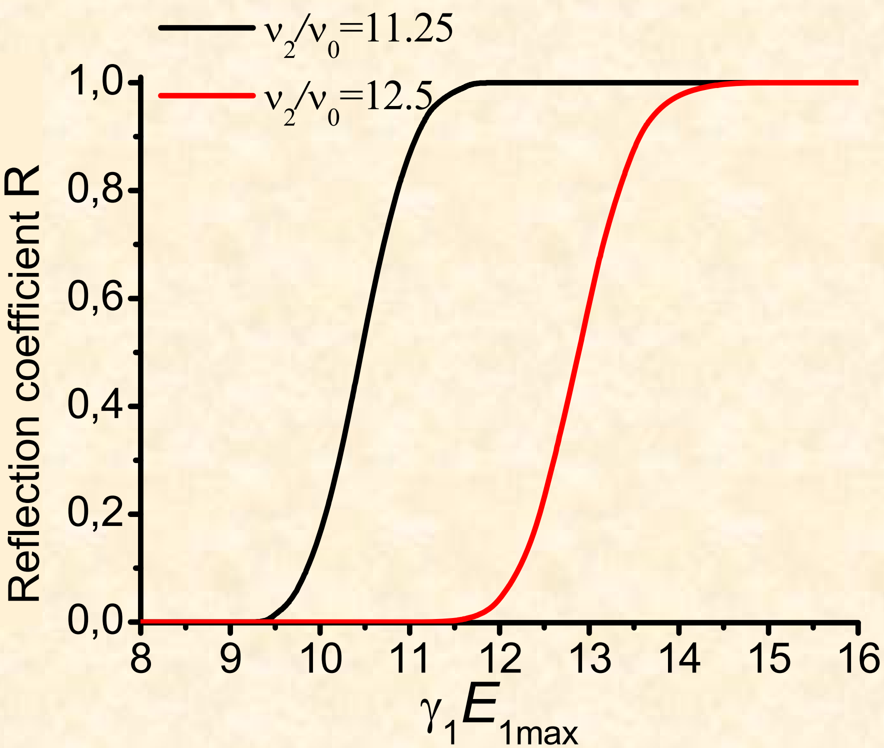
The results of numerical simulation of wave equations describing the two-pulse collision

- The trajectories of the signal pulse z (t) when GVM is larger (green line) or smaller (red line) the critical value corresponding to total internal reflection.

Signal pulse reflection coefficient

- Dependence of reflection factor on GVM
- Dependence of reflection factor on pump amplitude



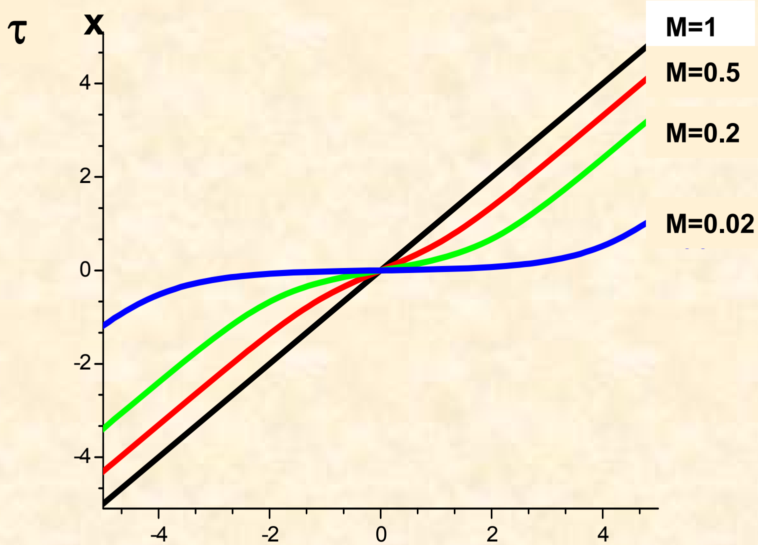


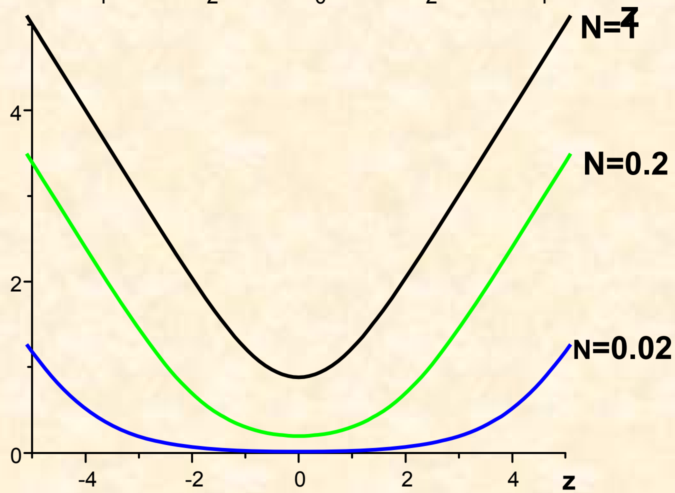
Near TIR pulse (beam) tunneling takes place

Trajectories of the signal pulses

In media with the inhomogeniety $n_{nl} = n_{nl}(0)\,ch^{-2}(\tau)$ the exact solutions for signal pulse tracing exist.

1 case. GVM is large: $\nu_0 > \nu_{cr}$





$$sh(\tau) = M\, sh(\nu_0 z);\;\; M = \sqrt{1-\nu_{cr}^2/\nu_{20}^2}$$

In linear media with $n_{nl} = 0,\ \nu_{cr} = 0,\ M = 1$ pulse travels along a straight line

$$sh(\tau) = sh(z);\;\; \tau = \nu_0 z$$

The trajectory of the signal pulse with the total reflection is given by

$$sh(\tau/T) = N\, ch(\nu_0 z/T)$$

$$N = \sqrt{\nu_{cr}^2/\nu_{20}^2 - 1}$$

The critical GVM equls to

$$\nu_{cr} = \sqrt{4k_{20} D_2 n_{nl}(0)} \approx 10^{-12}\, sm/s$$

Critical group velocity mismatch in different media

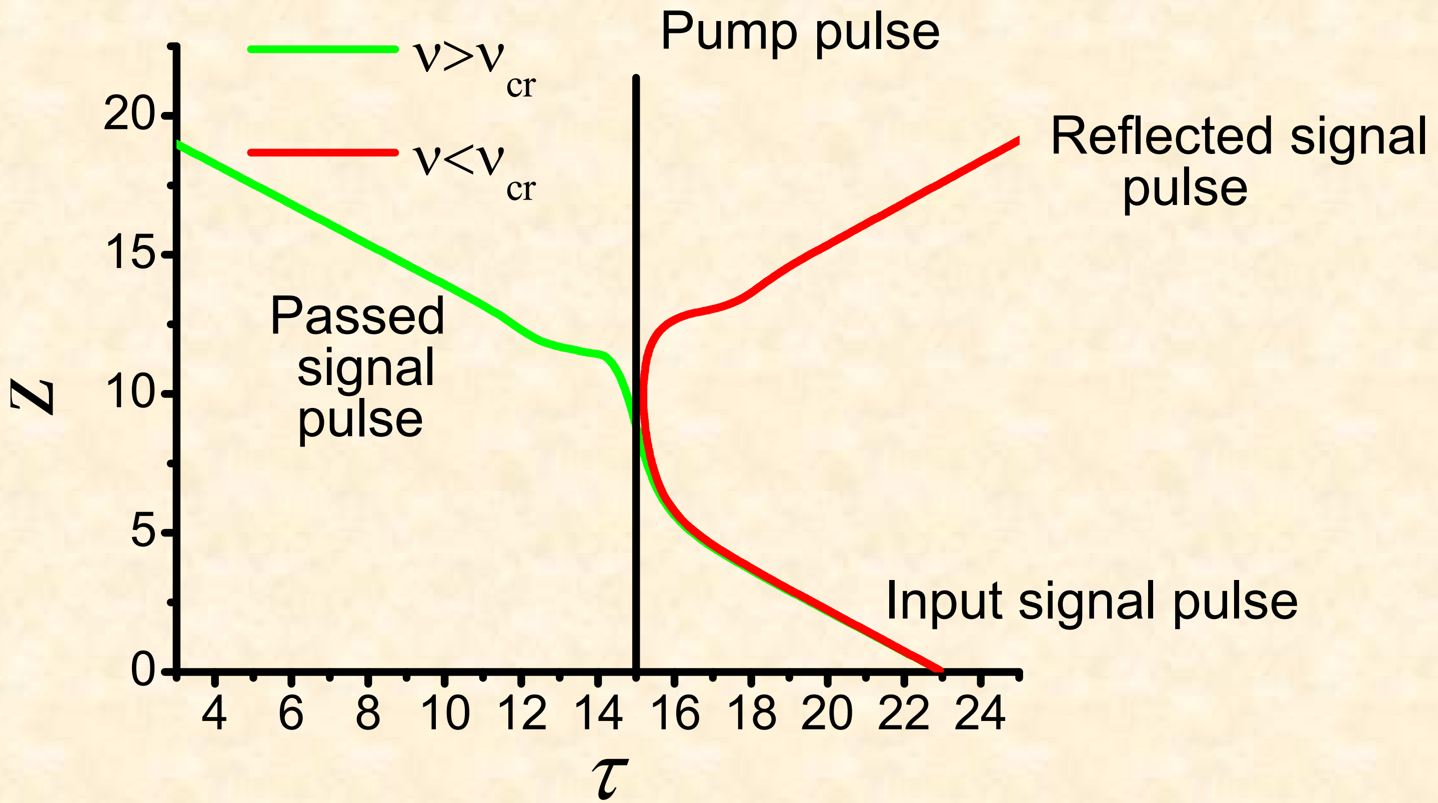
- Cascaded quadratic nonlinearity

$$\nu_{cr} = \left(4\gamma_2\gamma_3 \left|A_{1\max}\right|^2 \left|D_2/\Delta k\right|\right)^{1/2}$$

- Cubic (Kerr-type) nonlinearity

$$\nu_{cr} = \left(4\gamma_2 \left|A_{1\max}\right|^2 \left|D_2\right|\right)^{1/2}$$

Numerical simulation of signal and basic pulses propagation in a dispersive nonlinear media (wave theory)

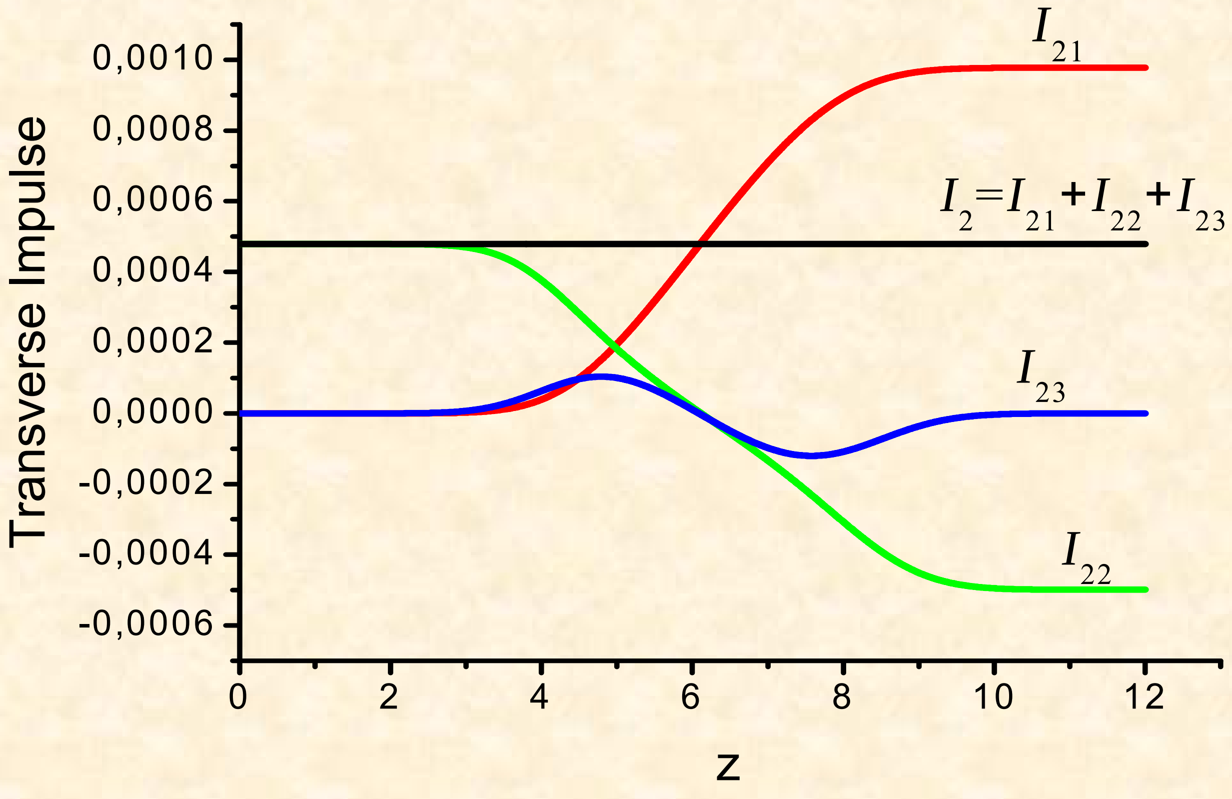


- Pulse trajectory $z = z(\tau)$: GVM is (green curve) more and (red) less than the critical mismatch, corresponding the total internal reflection.

PUMP BEAM RECOIL

TRANSVERSE IMPULSE CONSERVATION LAW

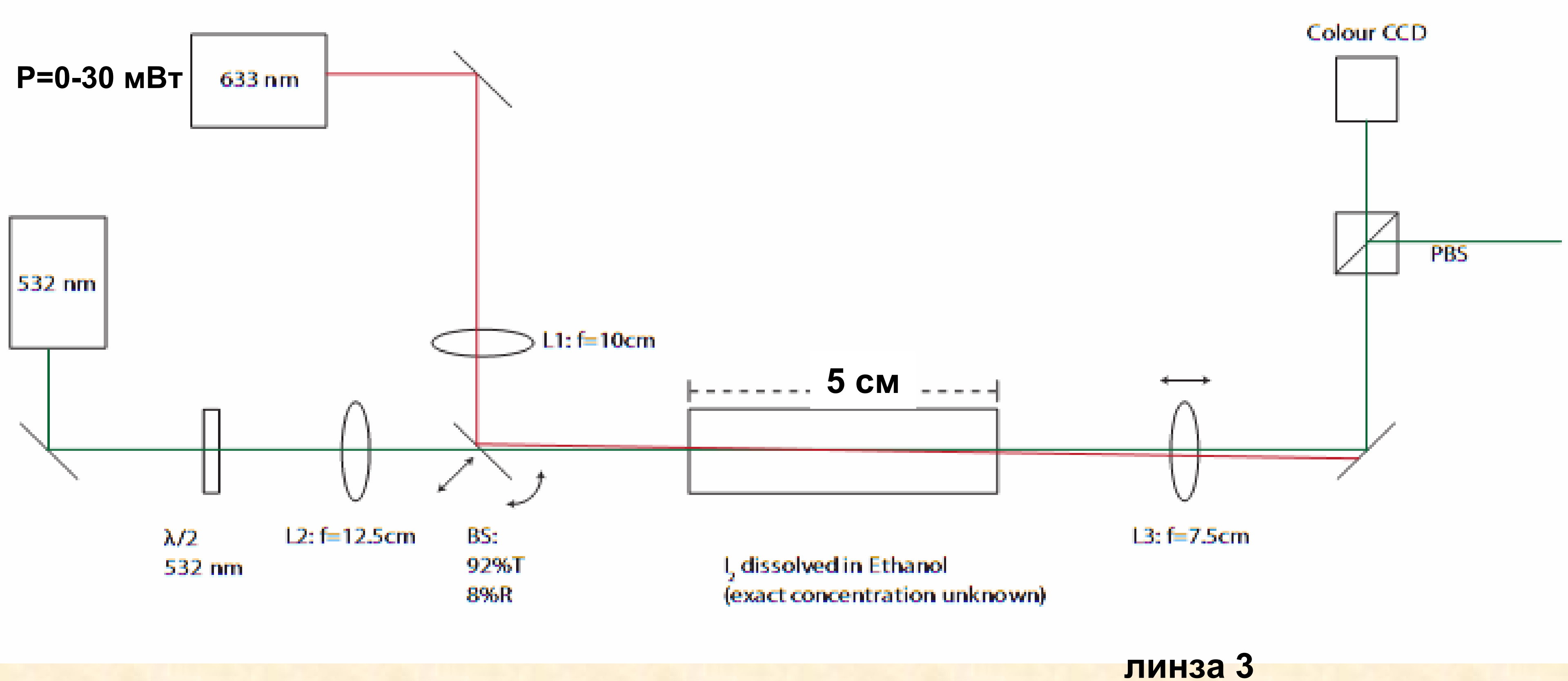
$$I_2 = \mathrm{Im}\int \sum_{n=1}^{3} \frac{1}{\gamma_n} A_n^* \frac{\partial A_n}{\partial x} dx = \int \sum_{n=1}^{3} \frac{1}{\gamma_n} \left| A_n^2 \right| \frac{\partial \varphi_n}{\partial x} dx = const$$



$$\theta_1 = \frac{2P_{signal}}{P_{pump}} \theta_2$$

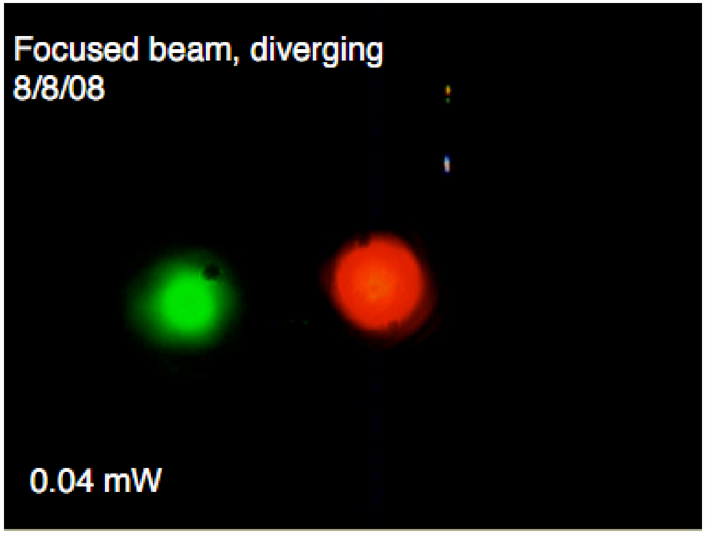
Pump beam is deflected to the opposite side after collision.

Total internal reflection in defocusing liquid

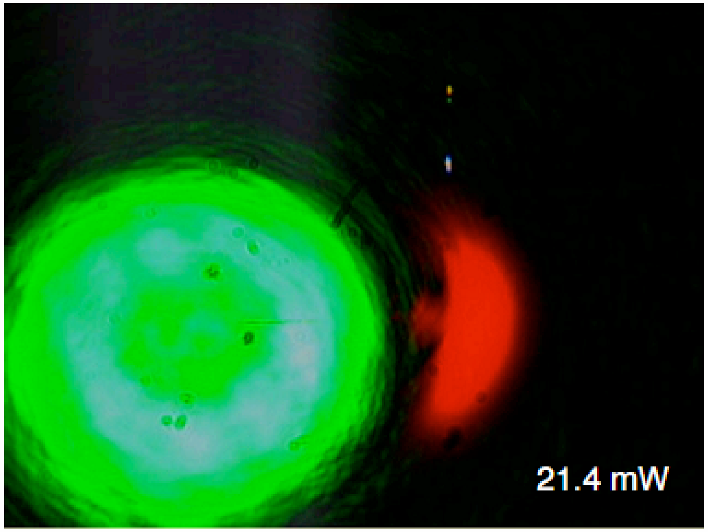


Experimental set-up

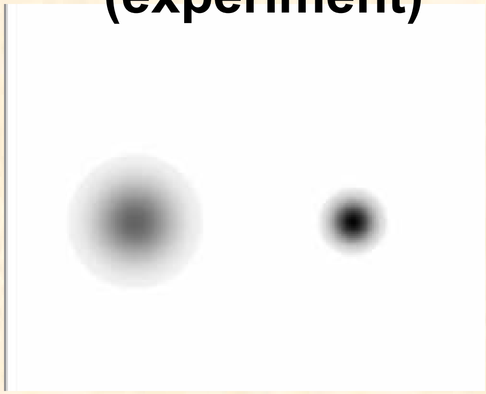
Total internal reflection in defocusing liquid



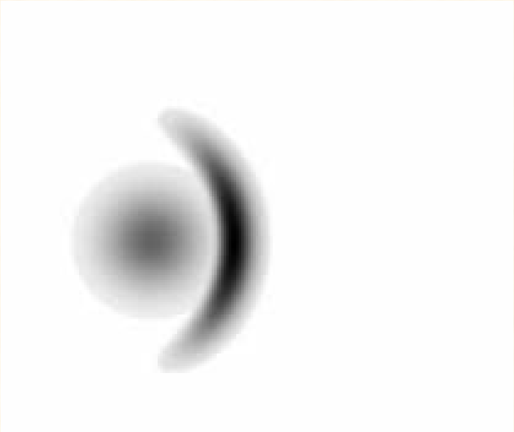
Θ=0.008



OutputcCross-sections of signal (red) and pump (green) laser beams (experiment)



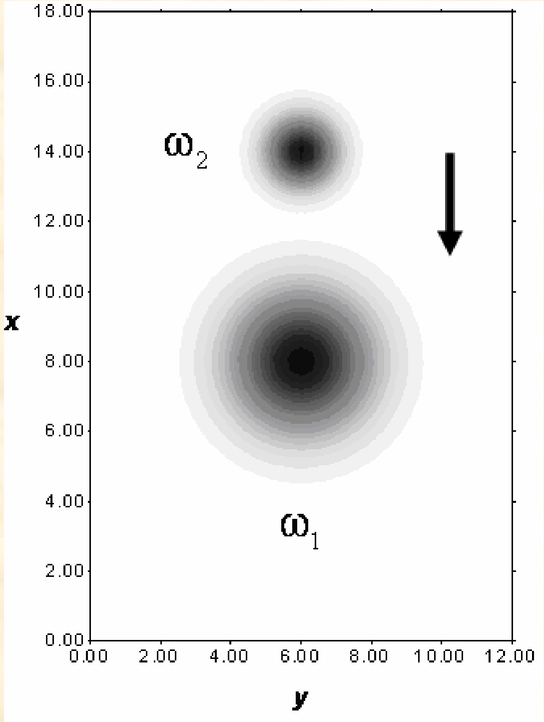
θ_c=0.01

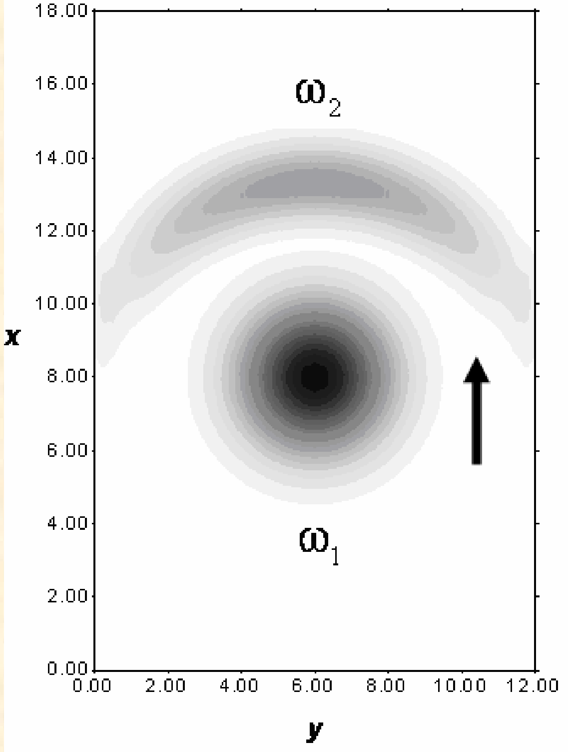


Numerical simulation

PARAMETRIC CONVEX MIRROR EFFECT

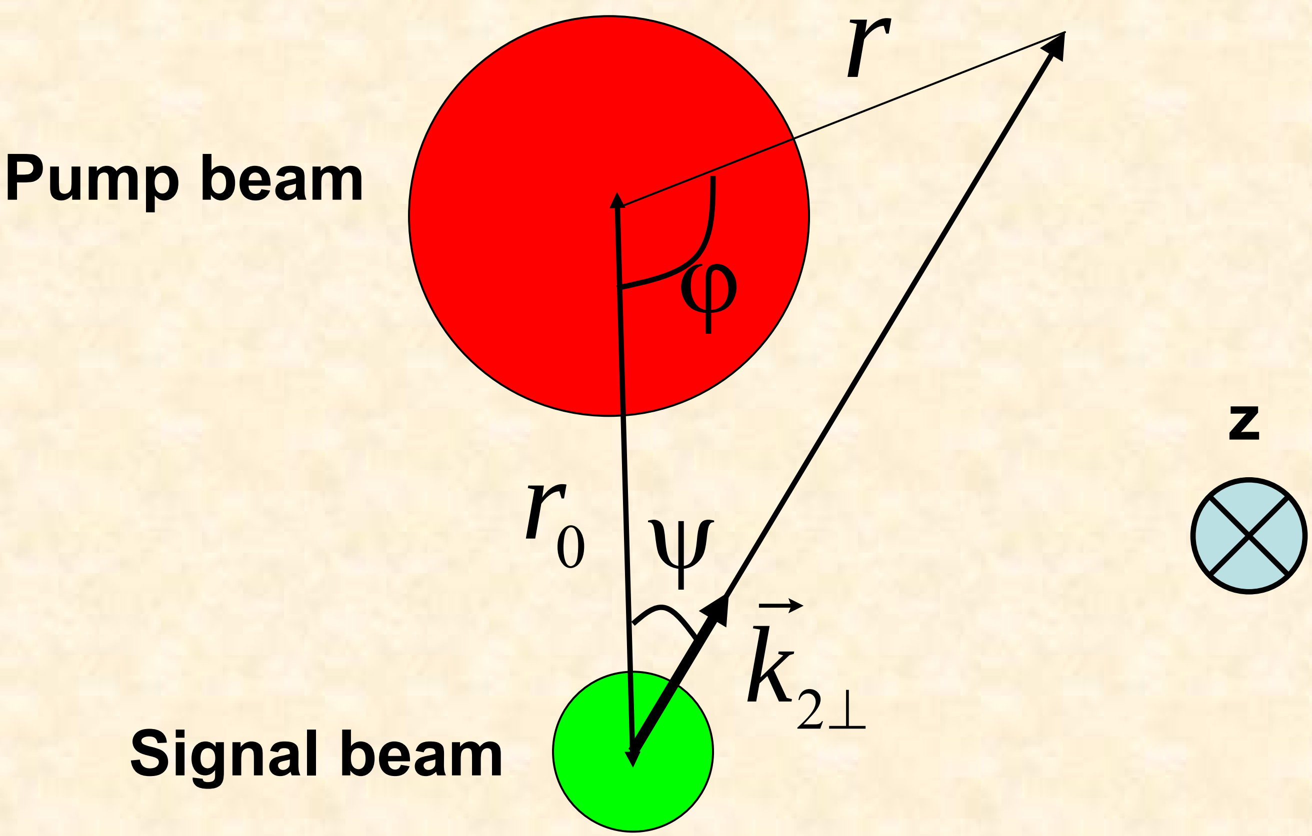
The pump beam shape influences on reflected signal beam divergence as $n_{nl} \sim \left|E_1(x,y)\right|^2$.





The incident beam (top) **Reflection from Gaussian pump beam**

NONCENTRAL COLLISION OF OPTICAL BEAMS



$\psi = 0$ **- central collision**

Transverse sections of beams

3-D SIGNAL BEAM TRAJECTORIES

$$\frac{dr}{dz} = \pm\sqrt{2}\sqrt{\frac{\theta_2^2}{2}\left(1 - a^2/r^2\right) - n_{nl}(r) + n_{nl}(r_0)}$$

$$\frac{d\varphi}{dz} = a\,\theta_2/r^2$$

$a = r_0 \sin\psi$ **is the aiming parameter**

Signal beam propagation resembles beam scattering in central force field.

$r^2 \dfrac{d\varphi}{dz} = \text{const}$ **is the impulse momentum conservation law for quasiparticle in central field**

3-D SIGNAL BEAM TRAJECTORIES

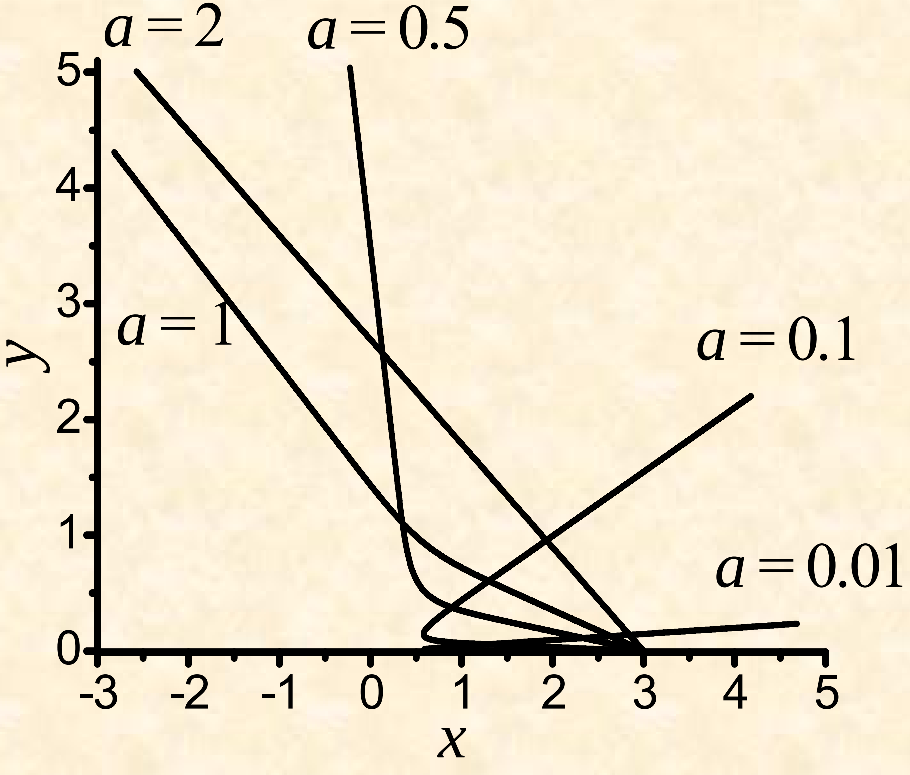
Signal beam propagation resembles beam scattering in central force field.

$$\frac{dr}{dz} = \pm\sqrt{2}\sqrt{\frac{\theta_2^2}{2}\left(1 - a^2/r^2\right) - n_{nl}(r) + n_{nl}(r_0)}$$

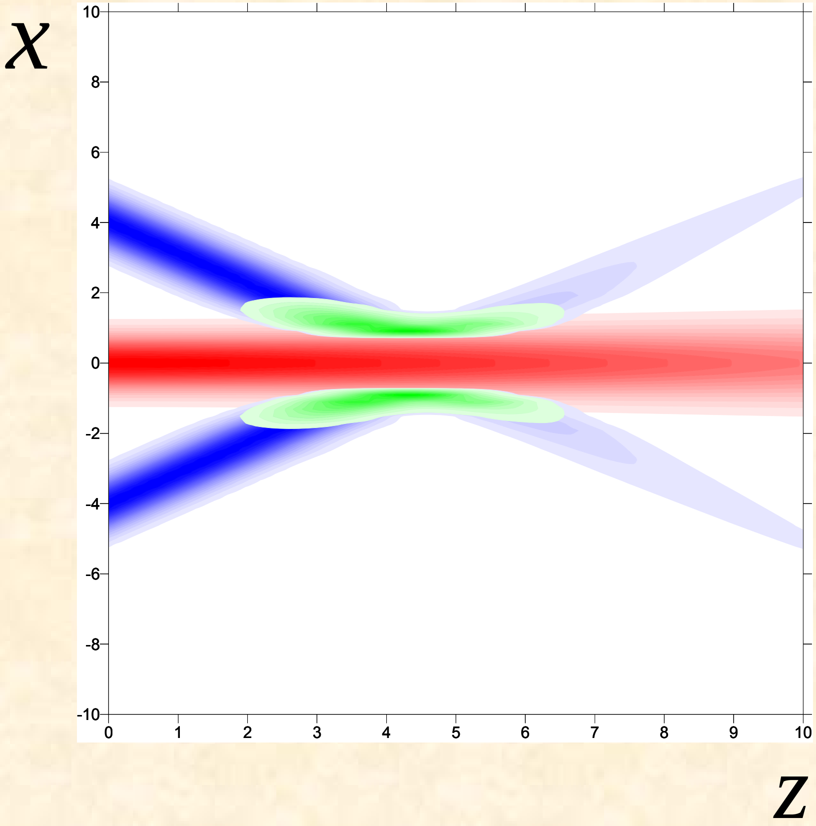
$$\frac{d\varphi}{dz} = a\theta_2/r^2$$

Aiming parameter $a = r_0 \sin\psi$

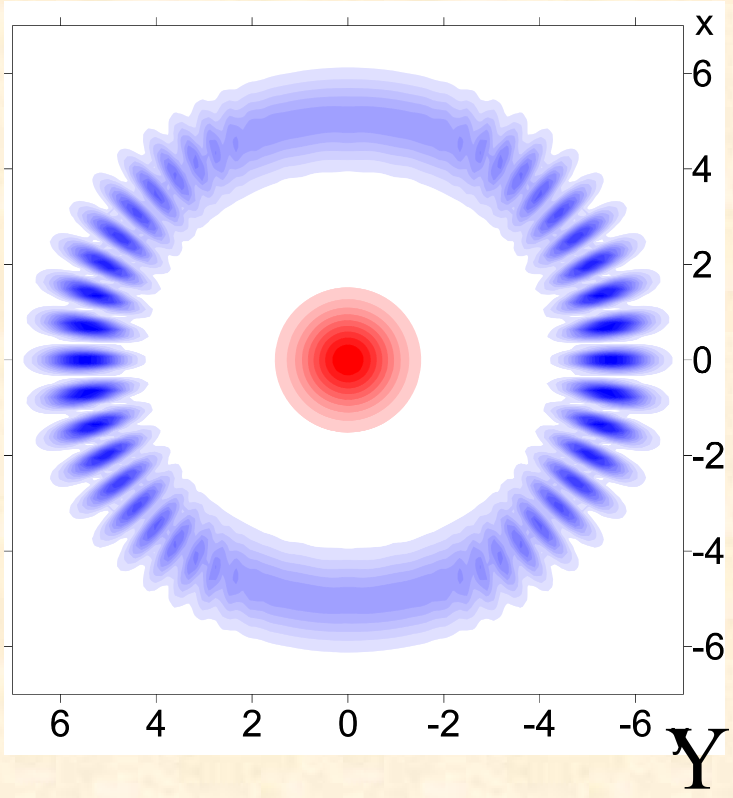
- **Signal beam trajectories for different aiming parameters**



INTERFERENCE PATTERN CREATED BY TWO SIGNAL BEAMS REFLECTED FROM PARAMETRIC MIRROR (PUMP BEAM) $a_1 = a_2$

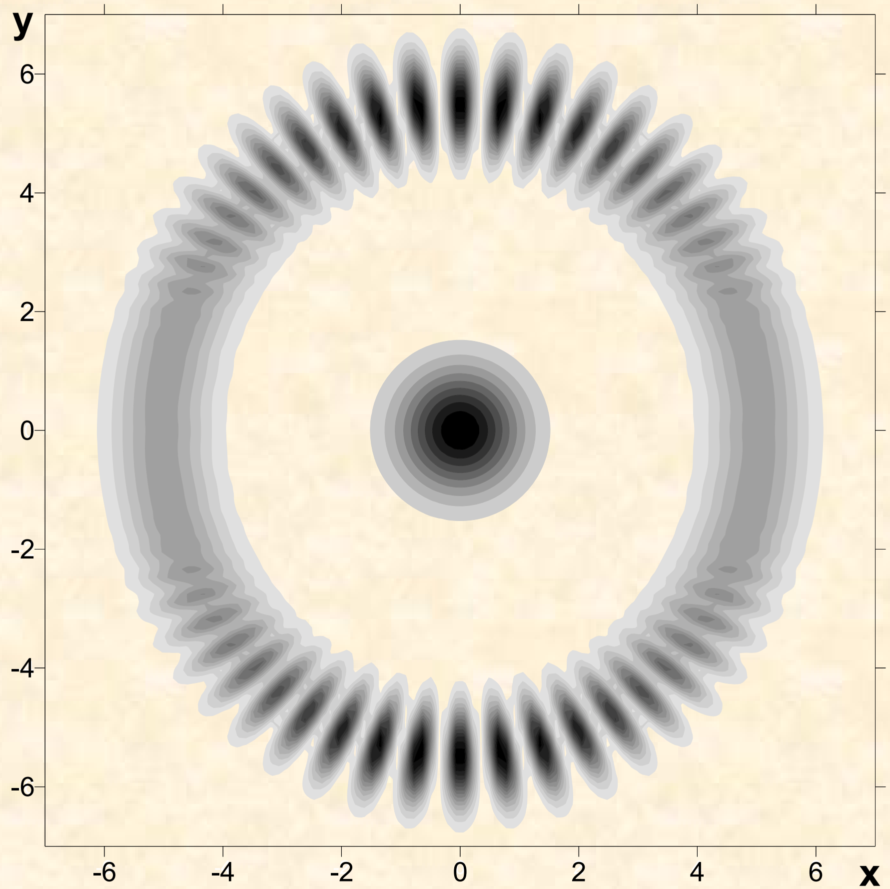


Longitudinal section of beams



Output cross-section of beams

INTERFERENCE PATTERN CREATED BY TWO SIGNAL BEAMS REFLECTED FROM PARAMETRIC MIRROR (PUMP BEAM)



Frequency shift effect

$$\Omega(z) = \frac{\partial S_0}{\partial \tau}$$

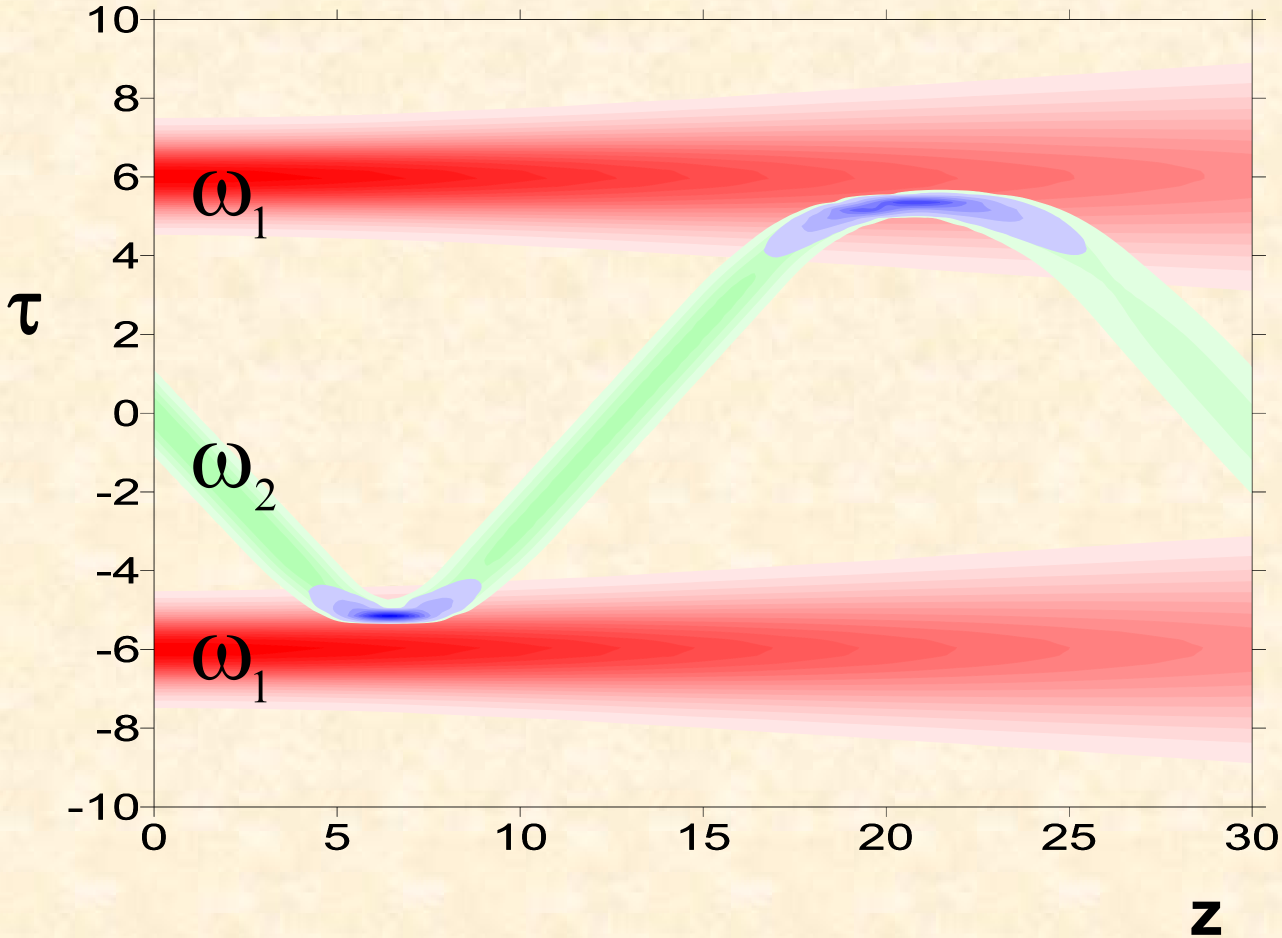
$$\Omega(z) = -\frac{\nu_2}{2D_2} \pm \sqrt{\left(\frac{\nu_2}{2D_2}\right)^2 + \frac{k_2[n_{nl}(\tau) - n_{nl}(\theta)]}{D_2}}$$

$$\Omega(0) = 0 \longrightarrow \Omega(z_{ref}) = -\frac{\nu_2}{D_2}$$

- As a result of frequency shift the signal velocity decreases and, as though, there is a delay of an signal pulse in comparison to a linear mode of propagation.
- *Change of velocity, frequency shift, and and delay time.*
- *Future experiments in fibers and PhC.*

Trapping signal pulse between two basic pulses

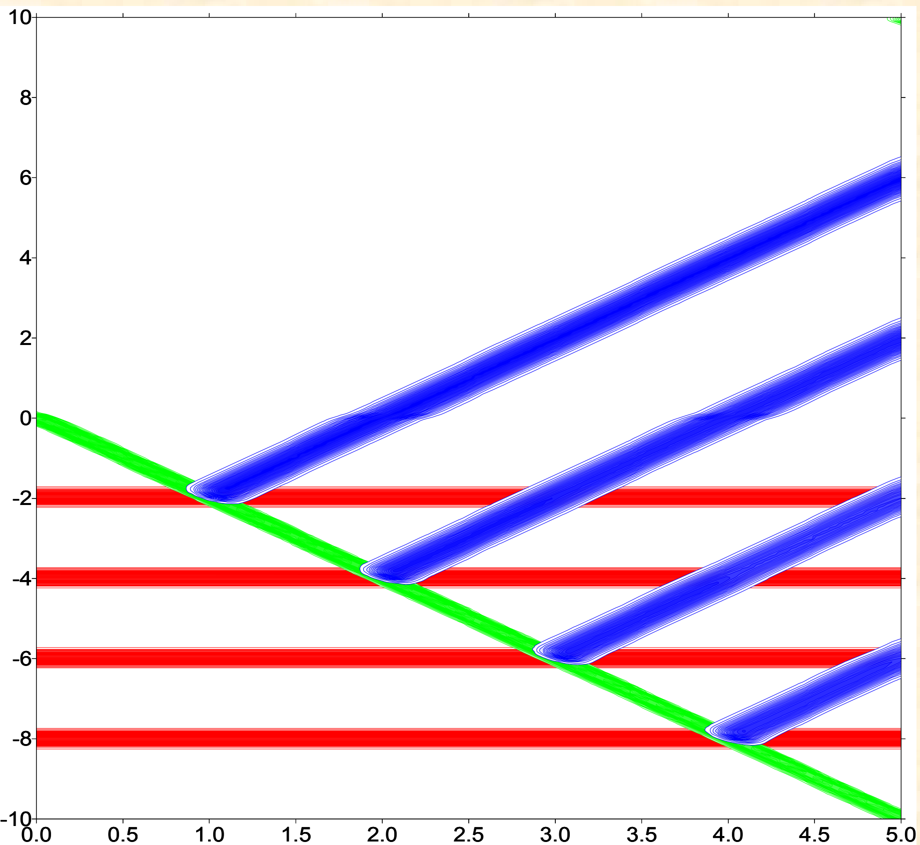
- The weak signal pulse propagates between two pump pulses, being serially reflected from them.

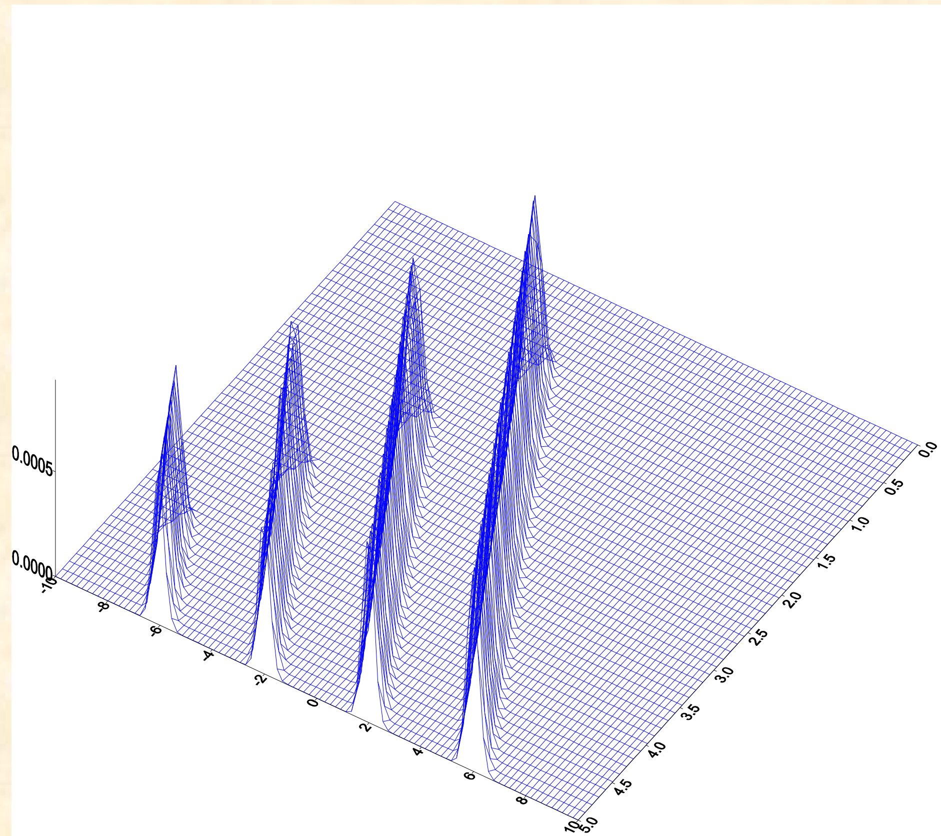


Reflected beam multiplexing by several pump beams

$$G_l = 0.77$$

$$d = 5a_1$$





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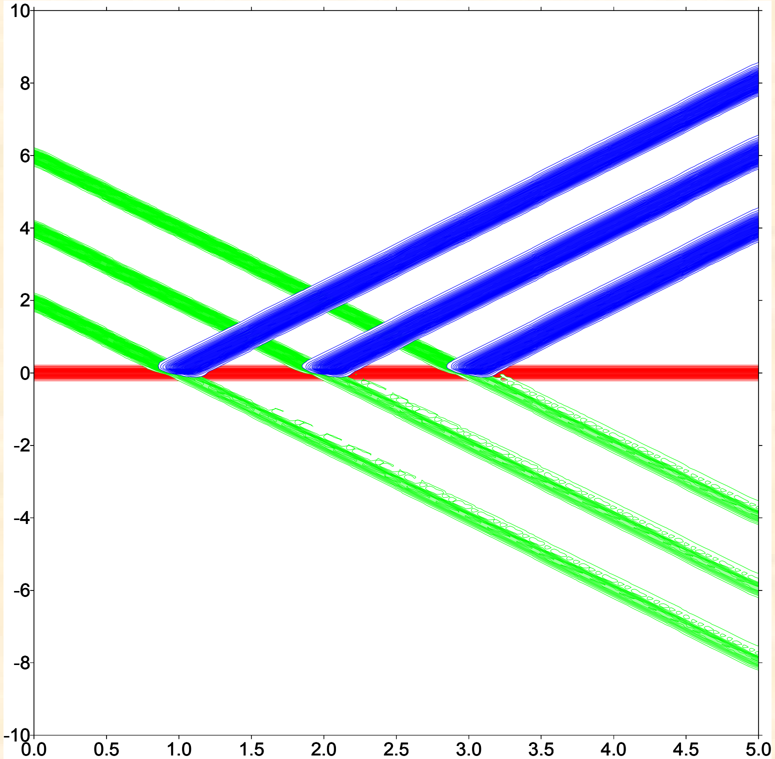
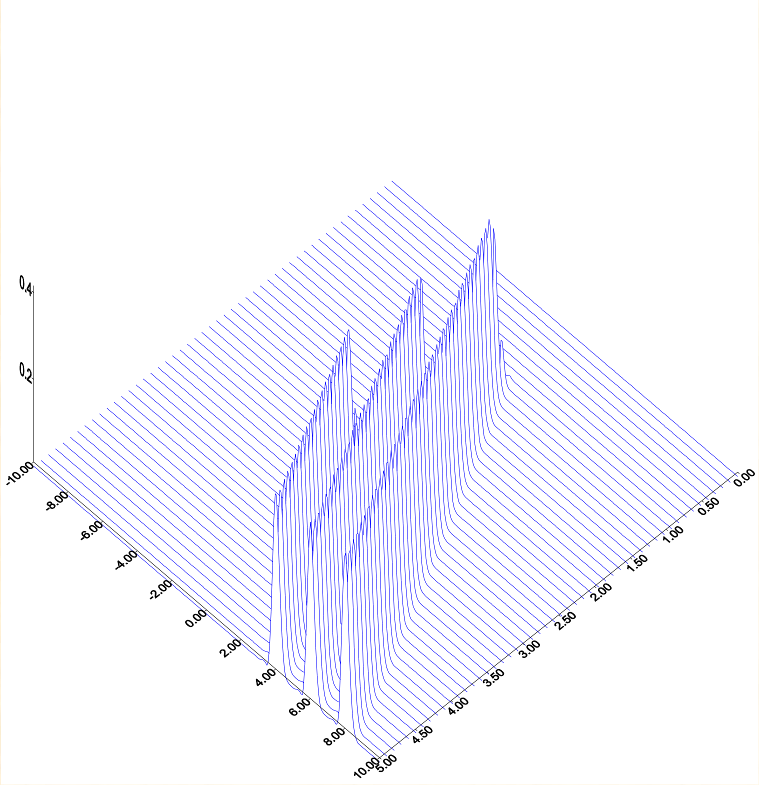
Reflected beam multiplexing by several incident signal beams

$$G_m = 4$$





Thank you for your attention